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Brief paper

Robust state estimation and unknown inputs reconstruction for a class of nonlinear systems: Multiobjective approach☆

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ABSTRACT

We consider a novel method to design H_∞ observers for a class of uncertain nonlinear systems subject to unknown inputs. First, the main system dynamics are rewritten as an augmented system with state vector including both the state vector of the main system and the unknown inputs. Then, we design a H_∞ reduced-order observer to estimate both state variables and unknown inputs simultaneously. Based on a Lyapunov functional, we derive a sufficient condition for existence of the designed observer which requires solving a nonlinear matrix inequality. To facilitate the observer design, the achieved condition is formulated in terms of a set of linear matrix inequalities (LMI). By extending the proposed method to a multiobjective optimization problem, the maximum bound of the uncertainty and the minimum value of the disturbance attenuation level are found. Finally, the proposed observer is illustrated with an example.

1. Introduction

Observer design for nonlinear systems is a popular problem in control theory that has been studied in many aspects. Moreover, state estimation of nonlinear system in the presence of unknown inputs is another fascinating and relevant topic in the modern control theory. We recommend our readers to see Chen and Saif (2006), Darouach (2009), Darouach, Zasadzinski, Onana, and Nowakowski (1995), Delshad (2015), Delshad and Gustafsson (2014), Ha and Trinh (2004) and Orjuela, Marx, Ragot, and Maquin (2009). However, there are not so many research efforts that have been done on reconstructing the unknown inputs. The main motivation behind an unknown input reconstruction is that in some applications it is either a costly task or not basically doable to measure some of the inputs (Park & Stein, 1988; Tu & Stein, 1996). On the other hand, an unknown input can represent the impact of failure of actuators or plant components and thus worth to be reconstructed and used in the field of fault detection and isolation (Patton & Chen, 1997; Zhang & Ding, 2007). In Corless and Tu (1998), authors proposed a combined state and input estimator which is applicable under very strict conditions. Xiong and Saif (2003) tried to extend (Corless & Tu, 1998) in a different way where the design is free of any necessity to be within the bounds of the unknown input. Moreover, Kim and Rew (2013) is another recent work which focuses on similar system dynamics in the discrete-time domain assuming slowly time-varying unknown inputs. It is worth to know that all the aforementioned works have been done on linear systems and not all have considered robustness of their method in a way we do in this paper. The purpose of the present paper is to propose a new observer design dealing with a class of nonlinear systems including uncertainty and also subject to unknown inputs, process and measurement noise. The proposed approach is based on the one developed in Darouach, Boutat-Baddas, and Zerrougui (2011) and Delshad, Johansson, Darouach, and Gustafsson (2014), where in Darouach et al. (2011) only the state variables are estimated for a singular system; here we also focus on unknown input reconstruction for a typical state space realization. In Delshad et al. (2014), authors consider the same problem as we do here; However, we allow uncertainty in the system dynamics and also we focus on a multiobjective optimization issue as the main contribution of this paper. It should be noted that there is some well-known research, for instance (Germani, Manes, & Palumbo, 2002), where the proposed approaches require the system dynamics to have strictly less number of unknown inputs than measurement outputs. In

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comparison with those works, our algorithm is more flexible in the sense that it only requires the number of unknown inputs to be less than or equal to the number of outputs. This paper also considers the problem of robustness. On one hand, the proposed observer design is robust in the sense that it shows a good performance of estimating the state variables of the system in the presence of unknown input. On the other hand, it minimizes the impact of any exogenous disturbance on the observer performance. Moreover, our proposed observer can handle parametric uncertainties in the system dynamics under some conditions. To the authors' best knowledge, this kind of analysis of robustness for the class of nonlinear Lipschitz system is quite new and not fully investigated. See for instance Abbaszadeh and Marquez (2009), Delshad and Gustafsson (2015) and Delshad, Gustafsson, and Johansson (2012) where authors have used a similar approach but for different problems. On the way of observer design, a sufficient condition for existence of the designed observer is derived, which requires solving a nonlinear matrix inequality. In order to facilitate the observer design, the obtained condition is formulated in terms of a set of linear matrix inequalities (LMI). We go one step further and convert the problem to a multiobjective optimization problem for finding the maximum bound of the uncertainties and minimum disturbance tuning parameter, simultaneously.

The rest of this paper is organized as follows. In Section 2, we introduce a class of nonlinear systems subject to unknown inputs. In Section 3, we propose a new method to design a reduced-order observer for the systems under study. The obtained nonlinear inequalities in Section 3 are then formulated as LMIs to facilitate the observer design. The robustness issue is discussed in Section 4. Later on and in Section 5, we convert the problem to a multiobjective optimization problem. To validate the proposed nonlinear observer, an example is presented in Section 6. Conclusions and future work are provided in Section 7.

2. Preliminaries and problem statement

Consider the following nonlinear system subject to unknown inputs:

$$\begin{aligned}\dot{x} &= Ax + Bu + f(x) + F\nu + D_1\omega,\\ y &= Cx + G\nu + D_2\omega\end{aligned} \tag{1}$$

where $x \in R^n$, $u \in R^m$, $\nu \in R^h$ and $y \in R^p$ are the state vector, known input, unknown input, and output vector, respectively. Matrices A, B, C, D_1, D_2, F and G are real and with appropriate dimensions. The function f is nonlinear and assumed to be differentiable. $\omega \in R^{n_\omega}$ is an exogenous disturbance containing both system and measurement noise. To specify the class of nonlinear systems under study, we use the following assumption.

Assumption 1. The function $f(x)$ is nonlinear and satisfies Lipschitz constraint as follows:

$$\|\Delta f\| = \|f(\sigma_1) - f(\sigma_2)\| \le \gamma\|\sigma_1 - \sigma_2\| \quad \forall \sigma_1, \sigma_2 \in R^n \tag{2}$$

where $\|\cdot\|$ is the 2-norm and $\gamma > 0$ is the Lipschitz constant.

3. Main results

Let $\zeta = \begin{bmatrix} x^T & \nu^T \end{bmatrix}^T$, then the augmented system dynamics are,

$$\begin{aligned}\mathscr{E}\dot{\zeta} &= \bar{A}\zeta + Bu + f(\mathscr{E}\zeta) + D_1\omega\\ y &= \bar{C}\zeta + D_2\omega\end{aligned} \tag{3}$$

where $\mathscr{E} = \begin{bmatrix} I_{n\times n} & 0_{n\times h} \end{bmatrix}$, $\bar{A} = \begin{bmatrix} A & F \end{bmatrix}$ and $\bar{C} = \begin{bmatrix} C & G \end{bmatrix}$.

We can make the following assumption which will be used in the sequel of the paper.

Assumption 2. *rank* $\begin{bmatrix} \mathscr{E} \\ \bar{C} \end{bmatrix} = n + h$.

Now, the aim is to design a robust observer such that it can estimate ζ asymptotically. Consider the following reduced-order observer for the augmented system (3),

$$\begin{aligned}\dot{z} &= Nz + Ly + Gu + Mf(\mathscr{E}\hat{\zeta})\\ \hat{\zeta} &= Jz + Ey\end{aligned} \tag{4}$$

where $z \in R^{n-p+h}$ and $\hat{\zeta}$ is the estimate of ζ. The matrices N, L, G, M, J, and E must be determined such that the error dynamics $e = \hat{\zeta} - \zeta$ converge to zero asymptotically. By defining the error between z and $M\mathscr{E}\zeta$,

$$\varepsilon = z - M\mathscr{E}\zeta \tag{5}$$

the error dynamics become,

$$\begin{aligned}\dot{\varepsilon} =& N\varepsilon + (NM\mathscr{E} + L\bar{C} - M\bar{A})\zeta + (G - MB)u\\ &+ M(f(\mathscr{E}\hat{\zeta}) - f(\mathscr{E}\zeta)) + (LD_2 - MD_1)\omega.\end{aligned} \tag{6}$$

On the other hand,

$$\begin{aligned}\hat{\zeta} &= Jz + Ey\\ &= J\varepsilon + (JM\mathscr{E} + E\bar{C})\zeta + ED_2\omega.\end{aligned} \tag{7}$$

Now, if there exists a matrix M such that,

$$G = MB, \qquad NM\mathscr{E} + L\bar{C} - M\bar{A} = 0, \qquad JM\mathscr{E} + E\bar{C} = I \tag{8}$$

or,

$$G = MB, \qquad \begin{bmatrix} N & L \\ J & E \end{bmatrix}\begin{bmatrix} M\mathscr{E} \\ \bar{C} \end{bmatrix} = \begin{bmatrix} M\bar{A} \\ I \end{bmatrix} \tag{9}$$

then (6) and (7) become,

$$\begin{aligned}\dot{\varepsilon} &= N\varepsilon + M\Delta f + (LD_2 - MD_1)\omega\\ e &= J\varepsilon + ED_2\omega\end{aligned} \tag{10}$$

where $\Delta f = f(\mathscr{E}\hat{\zeta}) - f(\mathscr{E}\zeta)$. We will now formulate and solve an H_∞ observer design problem with the following conditions:

(1) The observer error (10) with $\omega = 0$ is stable.
(2) Under zero initial condition, $\|e\|_2 < \mu\|\omega\|_2$.

3.1. Stability analysis

Since $e = J\varepsilon$ for $\omega = 0$, obviously the asymptotic stability of ε is sufficient condition for $\lim_{t\to\infty} e(t) = 0$. The following theorem gives conditions for stability of $e(t)$.

Theorem 1. *Consider the system (1) together with the nonlinear observer (4). Assume that, given admissible Lipschitz constant $\gamma > 0$ (satisfying Assumption 1) and for $\omega = 0$, if there exist real matrices N, J, M, and $P > 0$ with appropriate dimensions (satisfying (9)) such that the inequalities below are satisfied,*

$$\Gamma - I < 0, \qquad \begin{bmatrix} N^TP + PN + \gamma^2 J^TJ & PM \\ M^TP & -\Gamma \end{bmatrix} < 0 \tag{11}$$

then the state estimation error (10) produced by observer (4) tends to zero asymptotically.

Proof. First, consider a Lyapunov functional of $V = \varepsilon^T P\varepsilon$; where P is a positive definite matrix. From (10), Assumption 1, and by taking the derivative of $V(t)$ along the trajectory of (10) for $\omega = 0$, we have

$$\begin{aligned}\dot{V} =& \dot{\varepsilon}^T P\varepsilon + \varepsilon^T P\dot{\varepsilon}\\ =& [N\varepsilon + M\Delta f]^T P\varepsilon + \varepsilon^T P[N\varepsilon + M\Delta f]\end{aligned} \tag{12}$$