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Brief paper

Switched/time-based adaptation for second-order sliding mode control☆

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ABSTRACT

Controlling highly uncertain nonlinear systems is in general a quite difficult task, for which Sliding Mode (SM) control has proved to be an effective option. This brief proposes a SM control strategy which combines a switched policy with a time-based adaptation of the control gain, thereby allowing to effectively deal with a very conservative prior knowledge of the upper bounds on the uncertainties, that usually leads to a large control authority, and related performance degradation. With the proposed approach, the control effort is adjusted online according to the actual magnitude of the uncertain terms, eliminating the conservatism in the selection of the control gain.

1. Introduction

Sliding mode (SM) control has long been recognized as a powerful control method to counteract non-vanishing external disturbances and unmodelled dynamics (Utkin, Guldner, & Shi, 1999), these uncertainty sources being usually time-dependent, highly unpredictable and with arbitrary monotonicity. Yet, in conventional SM control, design relies on the knowledge of worst-case upper bounds of the uncertain terms, which, in most practical cases, result in being highly conservative, with an associated large control authority that may cause a non-negligible chattering. Several approaches to adapt the control effort in SM control have been proposed in the literature over the last decade, (see *e.g.*, Plestan, Shtessel, Br égeault, & Poznyak, 2010, Kochalummoottil, Shtessel, Moreno, & Fridman, 2012 and Shtessel, Moreno, Plestan, Fridman, & Poznyak, 2010), which make the amplitude of the control law track the magnitude of the uncertain terms but add significant complexity to the control scheme, along with the transient issues of traditional adaptive algorithms.

Recently, switched algorithms proved to be an efficient choice to achieve performance enhancement with a limited increase in the controller complexity, see *e.g.*, Corradini and Orlando (2002), Magni, Scattolini, and Tanelli (2008) and Tanelli and Ferrara (2013). With reference to SM control, Tanelli and Ferrara (2013) presented a switched formulation of second order sliding mode (S-SOSM) controllers, designing a different SOSM control law for each region of the state space in which specific uncertainty bounds are given. In this work, we aim at presenting an original combination between *switched* and *time-based* adaptation which allows us to manage the underlying control problem with unique and enhanced features as compared to the existing literature. Specifically, we only ask for very conservative guesses on the upper bounds on the uncertain terms, as the proposed online adaptation allows us to cope with such conservatism and retune the control gain to track the *actual* uncertainties. Therefore, we can directly compare with some of the adaptive methods that do not ask for knowledge of initial upper bounds, and offer – with respect to such purely adaptive solutions – all the advantages of the switched SM philosophy. At the same time, with respect to fixed-structure SM approaches where the controller parameters cannot vary with time, our solution allows to avoid the performance degradation and excessive control authority that comes from tuning the controller parameters based on poor knowledge on the upper bounds on the uncertainties,

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yielding unnecessarily high gains. A first proposal of the time-based switched adaptation strategy was presented in Bartolini, Levant, Pisano, and Usai (1999) and Capisani, Ferrara, and Pisano (2011). In the present paper, such a strategy is developed for the Suboptimal SOSM control algorithm, Bartolini, Ferrara, and Usai (1998), and designed jointly with the switched controller. The resulting switched/adaptive control law is shown to yield finite-time convergence to an invariant set, containing the origin, defined on the basis of the time-varying uncertainty. Most interestingly, the size of such an invariant set can be *a-priori* estimated based on the controller parameters and on the disturbance characteristics, so that it can be guaranteed to be contained in the innermost region of the state-space partition, so that the evolution of the two variables defining the SOSM dynamics is ultimately uniformly bounded. A preliminary and short version of this work, which did not contain the proofs of the algorithm convergence, was presented in Pisano, Tanelli, and Ferrara (2013).

The paper is organized as follows. Section 2 introduces the considered problem and working assumptions. Section 3 presents the proposed control algorithm and its convergence properties, while its performance is discussed in Section 4.

2. Problem statement

We deal with nonlinear, single-input single-output, uncertain nth order system that can be transformed into the so-called perturbed chain of integrators form (see *e.g.*, Dinuzzo and Ferrara (2009)), which takes the expression

$$\begin{aligned}\dot{x}_{i+1} &= x_i, \quad i = 1, \dots, n-1\\ \dot{x}_n &= \lambda(x(t)) + \rho(t) + d(x(t))u(t),\end{aligned} \tag{1}$$

with $x = [x_1, \dots, x_n]^T$ being the system state and $\lambda(x(t))$, $\rho(t)$ and $d(x(t))$ being sufficiently smooth nonlinear uncertain functions. Within a sliding mode control framework, assume to select a sliding manifold

$$s(x(t)) = x_n(t) + \sum_{i=1}^{n-1} c_i x_i(t) = 0, \tag{2}$$

with the c_i being positive constants that make the characteristic equation $z^{n-1} + \sum_{i=1}^{n-1} c_i z^{i-1} = 0$ has all roots with negative real part. Then it can be shown that (see Bartolini et al., 1998) if the sliding manifold can be reached in finite time using a second order SM controller with a discontinuous control signal $\dot{u}(t)$, once on the sliding manifold the system behaves as a reduced-order, asymptotically stable linear system.

Consider now the second order uncertain nonlinear system (often referred to as the "auxiliary" system)

$$\begin{aligned}\dot{z}_1 &= z_2\\ \dot{z}_2 &= f(z(t)) + h(t) + g(z(t))v(t),\end{aligned} \tag{3}$$

where $z(t) = [z_1(t) \ z_2(t)]^T \in \mathbb{R}^2$ is the system state, $z_1(t) = s(x(t))$ is the sliding variable, $v(t) = \dot{u}(t)$ is the control signal and $f(z(t))$ and $g(z(t))$ are uncertain, sufficiently smooth functions, satisfying all the conditions ensuring existence and uniqueness of the solution (Khalil, 1996), and with $h(t)$ being a time-varying perturbation for which a (possibly very conservative) upper bound H is known, *i.e.*,

$$|h(t)| \ll H, \quad \forall t \geq 0. \tag{4}$$

In the case where $h(t) = 0$, and it holds that

$$0 < G_1 \leq g(z(t)) \leq G_2, \qquad |f(z(t))| \leq F, \tag{5}$$

the Suboptimal control law (see *e.g.*, Bartolini et al., 1998)

$$\begin{aligned} v(t) &= -\alpha V \,\mathrm{sign}\Big(z_1(t) - \beta z_{\mathrm{Max}}\Big), \quad \beta = \frac{1}{2}\\ \alpha &= \begin{cases}\alpha^* & \text{if} \quad [z_1(t) - \beta z_{\mathrm{Max}}][z_{\mathrm{Max}} - z_1(t)] > 0\\ 1 & \text{else,}\end{cases}\end{aligned} \tag{6}$$

where V is the control gain, α is the so-called modulation factor, and z_{Max} is a piecewise constant function representing the value of the last *extremal* point of $z_1(t)$ (an extremal or singular point of the trajectory is defined, see *e.g.*, Bartolini, Ferrara, and Usai (1997) and Boiko, Fridman, Pisano, and Usai (2007), as a local minimum, local maximum, or a horizontal flex point) makes the system trajectory converge onto the sliding manifold $z_1 = z_2 = 0$ in finite time provided that the control parameters α^* and V are chosen so as to satisfy

$$\alpha^* \in (0, 1] \cap \left(0, \frac{3G_1}{G_2}\right), \qquad V > \max\left\{\frac{F}{\alpha^* G_1}, \frac{4F}{3G_1 - \alpha^* G_2}\right\}. \tag{7}$$

In this work, the class of perturbations that can be dealt with by traditional SM control approaches is enlarged, and the auxiliary system is modified as in (3) allowing the presence of the additional time-varying perturbation $h(t)$, along with its associated, possibly very conservative, upper bound H. Note that in principle it would be possible to use the suboptimal algorithm with an oversized control magnitude V obtained according to (7) with the constant F being replaced by $F + H$. This approach, however, would lead to unacceptable chattering. Additionally, as in Tanelli and Ferrara (2013), we are going to consider region-dependent uncertainty bounds for the uncertain function $f(\cdot)$. Consider system (3) under the following assumptions.

(i) **State-space partitioning**: We assume that the state space $\mathcal{Z}$ of system (3) is partitioned into k regions $\mathcal{R}_i$, $i = 1, \dots, k$ defined as

$$\mathcal{R}_i := \left\{(z_1, z_2) : |z_1| \leq \bar{z}_{1,i} \text{ and } |z_2| \leq \bar{z}_{2,i}\right\}, \tag{8}$$

with $\bar{z}_{j,i-1} > \bar{z}_{j,i}, j = 1, 2, i = 2, \dots, k-1$, while the outermost region $\mathcal{R}_1$ is defined as

$$\mathcal{R}_1 := \left\{(z_1, z_2) : |z_1| \geq \bar{z}_{1,1} \text{ and } |z_2| \geq \bar{z}_{2,1}\right\}. \tag{9}$$

Further, define $\mathcal{W}_i = \partial\mathcal{R}_{i+1}$, $i = 1, \dots, k-1$. We introduce the regions $\mathcal{Z}_1 \equiv \mathcal{R}_1$, $\mathcal{Z}_i = \mathcal{R}_i \setminus \mathcal{R}_{i+1}$, $i = 2, \dots, k-1$ and $\mathcal{Z}_k \equiv \mathcal{R}_k$, which are such that $\cup_{i=1,\dots,k} \mathcal{Z}_i = \mathcal{Z}$, and we assume that in each of them different upper and lower bounds for the uncertainties can be defined, to be specified in the following. Note that only the innermost region $\mathcal{Z}_k$ contains the origin.

(ii) **State-dependent uncertainty description**: Let us assume that in each region $\mathcal{Z}_i$, $i = 1, \dots, k$ a constant upper bound on the uncertain terms is known, *i.e.*, $\forall\, i = 1, \dots, k$ one can write

$$0 < \overline{\mathcal{G}}_{1,i} \leq g(z(t)) \leq \overline{\mathcal{G}}_{2,i}, \qquad |f(z(t))| \leq \overline{\mathcal{F}}_i, \quad z \in \mathcal{Z}_i. \tag{10}$$

Such upper bounds can be determined owing on the fact that within each of such regions the state norm of the auxiliary system variables is bounded. In presence of the additional time-varying uncertainty affecting (3) no fine tuning of the amplitude of the control law can be made relying on the S-SOSM strategy only, and thus in this work we complement it with a time-based adaptation mechanism, which is introduced in the following section.

Remark 1. The assumption of constant bounds for the uncertain functions entering the auxiliary system can be relaxed by exploiting the results presented in Bartolini, Pisano, Ferrara, and Usai (2001), where such bounds were replaced by uncertain functions of the system state with linear growth, proving a semiglobal convergence result for the suboptimal algorithm with constant gain.