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Brief paper

Mode-identifying time estimation and switching-delay tolerant control for switched systems: An elementary time unit approach☆

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ABSTRACT

In this paper, an elementary time unit (ETU) method is proposed to study the problem of estimating the admissible delay in the identification of the active mode in the analysis and design of switched systems. The activation interval of a subsystem is considered to consist of a finite-time number of segments called ETUs, by which a novel class of time-scheduled Lyapunov function is used to estimate the admissible delay in mode identification for switched systems. Further, the ETU method is applied for switching-delay tolerant control problem, and a class of time-scheduled state feedback controllers are designed to achieve the exponentially stability. Several numerical examples are presented to validate the theoretic findings.

1. Introduction

Switched systems have emerged as an important class of hybrid systems representing a very active research area in the field of systems and control (Chesi, Colaneri, Geromel, Middleton, & Shorten, 2012; Dehghan & Ong, 2012; Duan & Wu, 2014; Geromel & Colaneri, 2006; Hu, Shen, & Zhang, 2011; Lee & Dullerud, 2007; Lin & Antsaklis, 2007, 2009; Lu, Wu, & Kim, 2006; Margaliot, 2006; Shorten, Wirth, Mason, Wulff, & King, 2007; Zhang, Hu, & Abate, 2012). Switched system can be efficiently used to model many practical systems that are inherently multi-model in the sense that several dynamical subsystem models are required to describe their behaviors, e.g. see Morse (1996). A basic issue in control of switched systems is the mode identification to implement mode-dependent controllers. The mode identification is necessary in practice, and the process with multiple operating modes has to be controlled by a set of multiple controllers along with a mode estimator. Therefore, to determine the admissible mode-identifying time for concrete switched systems will be of great significance as *a priori* information to evaluate different identification methods. However, note that the problem still remains largely open in the area.

In most of real applications, the value of switching signal $\sigma_p(t)$ is unavailable once the switching of process occurs. One would naturally try to first identify the switching signal at the beginning of each time interval using a short time period $[t_k, t_k+\tau)$ (generally $\tau \ll t_{k+1} - t_k$), e.g. some design results for mode estimator in Baglietto, Battistelli, and Tesi (2013), Battistelli (2013), and then control the identified system in the rest of the time interval. As what has been suggested in numerous articles (Mahmoud & Shi, 2012; Vu & Morgansen, 2010; Xiang, Xiao, & Iqbal, 2011; Zhang & Gao, 2010; Zhang & Shi, 2009), the very first and important concern is that the inappropriately large (inadmissible) mode-identifying time τ would turn the stable closed loop to be unstable since the correct controller cannot be activated in time. Thus, the problem arises here:

- **Problem A.** Given a set of feedback controllers, how to estimate the admissible mode-identifying time τ by which the stability of closed loop holds?

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Then, from the control point of view, designing a set of controllers stabilizing the closed loop in the presence of a mode-identifying time (producing switching delay) τ is of interest. Furthermore, the controller is expected to tolerate the switching delay as large as possible. Therefore, as an opposite to Problem A, the switching-delay tolerant control problem considered in this paper is:

- **Problem B.** How to design a set of stabilizing feedback controllers, which is capable of being tolerant to the switching delay τ as large as possible?

Due to the inevitable and necessary existence of mode-identifying process in actual applications, both of above problems are not only theoretically interesting and challenging, but also very important in practical applications, which motivates the present study in this paper.

Inspired by the discretized Lyapunov function approach widely used in time-delay systems (Gu, Kharitonov, & Chen, 2003), and also inspired by the recent articles for switched system with dwell time constraint (Allerhand & Shaked, 2011, 2013), the elementary time unit (ETU) approach is proposed to solve the problems presented in this paper. Briefly speaking, the ETU is the activation interval being classified by mode-identifying interval and normal-working interval, and both of which consist of a finite number of segments with different resolutions. It is worth mentioning that some previous results such as Mahmoud and Shi (2012), Zhang and Gao (2010), Zhang and Shi (2009) and Xiang et al. (2011), concerned with asynchronously switched system are covered by our ETU approach. The remainder of this paper is organized as follows. In Section 2, the considered systems and problems are formulated. In Section 3, the ETU technique is introduced and the mode-identifying time estimation is studied. The switching-delay tolerant control is studied in Section 4. Conclusions are given in Section 5.

Notations: Let $\mathbb{R}$ denote the field of real numbers, $\mathbb{R}_{\geq 0}$ stand for non-negative real numbers, and $\mathbb{R}^n$ be the n-dimensional real vector space. $\|\cdot\|$ stands for Euclidean norm. The notation $P \succ 0$ ($P \succeq$) means matrix P is real symmetric and positive definite (semi-positive definite). $P^\top$ denotes the transposition of matrix P and $\mathbf{He}\{P\} = P^\top + P$. Function int$[x]$ rounds the x to the nearest integer towards zero. $\xi_i(\cdot) : [0, \infty) \to \{0, 1\}$ are indication functions for switching signal $\sigma(t)$, it is defined as $\xi_i(t) = 1$ if $\sigma(t) = i$, otherwise $\xi_i(t) = 0$.

2. System description and problem formulation

In this paper, the switched system is in the form of

$$\dot{x}(t) = A_{\sigma_p(t)}x(t) + B_{\sigma_p(t)}u(t) \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}^m$ is the control input. Define index set $\mathcal{I} := \{1, 2, \ldots, N\}$ where N is the number of subsystems. $\sigma_p(t) : [0, \infty) \to \mathcal{I}$ denotes the switching signal, which is assumed to be a piecewise constant function continuous from right. $A_i \in \mathbb{R}^{n\times n}$ and $B_i \in \mathbb{R}^{n\times m}$ are known matrices of appropriate dimension. The discontinuities of $\sigma_p(t)$ are called *switches* and the switching sequence is expressed by

$$\mathcal{S}_p := \{(i_0, t_0), \ldots, (i_k, t_k), \ldots | i_k \in \mathcal{I}, k = 0, 1, \ldots\}$$

where t_0 denotes the initial time, t_k is the kth switching instant and $d_k = t_{k+1} - t_k$, $k = 0, 1, \ldots$. It is assumed that there exists a lower bound of d_k, i.e., $d_k \geq d_{\min} > 0$, $\forall k = 0, 1, \ldots$, where $d_{\min}$ is called minimal dwell time.

The mode-dependent controller is considered as

$$u(t) = K_{\sigma_c(t)}x(t) \tag{2}$$

where $K_i \in \mathbb{R}^{m\times n}$, for $\sigma_c(t) = i \in \mathcal{I}$ are constant matrix gains already designed. $\sigma_c(t) : [0, \infty) \to \mathcal{I}$ is the switching signal of controller. In the presence of mode-identifying process, the identification time is denoted as τ which directly leads to $\sigma_c(t) = \sigma_p(t - \tau)$. Trivially, it is assumed $0 < \tau < d_{\min}$. Hence, the switching sequence generated by $\sigma_c(t)$ can be expressed by

$$\mathcal{S}_c := \left\{(i_0, \hat{t}_0), \ldots, (i_k, \hat{t}_k), \ldots | i_k \in \mathcal{I}, k = 0, 1, \ldots\right\}$$

in which $\hat{t}_k = t_k + \tau < t_{k+1}, \forall k = 0, 1, \ldots$. Combining the sequences $\mathcal{S}_p$ and $\mathcal{S}_c$ and inspired by the idea in Vale and Miller (2011), each interval $[t_k, t_{k+1})$ can be essentially classified into the mode-identifying period $\mathcal{M}_1 := [t_k, t_k + \tau)$ with $\sigma_p(t) \neq \sigma_c(t)$, and the normal-working period $\mathcal{M}_2 := [t_k + \tau, t_{k+1})$ with $\sigma_p(t) = \sigma_c(t)$.

Let $\bar{A}_{i,j} := A_i + B_iK_j$ and $\mathcal{I}^2 := \mathcal{I} \times \mathcal{I}$, $\mathcal{I}^2_{\mathcal{M}_1}$ be the set of all indices in $\mathcal{I}^2$ such that $i \neq j, \forall i, j \in \mathcal{I}$, and $\mathcal{I}^2_{\mathcal{M}_2} := \mathcal{I}^2 \setminus \mathcal{I}^2_{\mathcal{M}_1}$ includes all indices such that $i = j, \forall i, j \in \mathcal{I}$. Substituting controller (2) into system (1) and considering the mode estimator, the dynamics of closed loop in interval $[t_k, t_{k+1})$ with $\sigma_p(t) = i, t \in [t_k, t_{k+1})$ and $\sigma_c(t) = j, t \in [t_k, t_k + \tau)$ can be derived as follows:

$$\dot{x}(t) = \begin{cases} \bar{A}_{i,j}x(t) & t \in [t_k, t_k + \tau),\ (i, j) \in \mathcal{I}^2_{\mathcal{M}_1} \\ \bar{A}_{i,i}x(t) & t \in [t_k + \tau, t_{k+1}),\ (i, i) \in \mathcal{I}^2_{\mathcal{M}_2} \end{cases} \tag{3}$$

where $0 < \tau < d_{\min}$ is the mode-identifying time.

Definition 1. System (3) is said to be exponentially stable with a decay rate $\beta > 0$ if $\|x(t)\| < Ce^{-\beta(t-t_0)}\|x(t_0)\|$ holds for any $x(t_0)$, any $t \geq t_0$ and a constant $C > 0$.

Then, given a mode-identifying time τ, the first problem is restated as follows.

Problem 1. Find sufficient conditions on the controller (2) for system (1) and on the mode-identifying time τ such that the closed loop (3) is exponentially stable.

When τ is uncertain, one could expect that an upper bound on it denoted by τ^*, below which the stability of the closed loop is guaranteed.

Problem 2. Given the controller (2) for system (1), estimate the upper bound of admissible mode-identifying time τ^* such that the exponential stability of closed loop (3) can hold for any $\tau \leq \tau^*$.

As to switching-delay tolerant control problem, the following time-varying controller covering (2) is considered

$$u(t) = \mathcal{K}_{\sigma_c(t)}(t)x(t) \tag{4}$$

where $\mathcal{K}_i(t), i \in \mathcal{I}$ are time-varying gain matrices to be determined. The closed loop system can be expressed by

$$\dot{x}(t) = \begin{cases} \bar{\mathcal{A}}_{i,j}(t)x(t) & t \in [t_k, t_k + \tau),\ (i, j) \in \mathcal{I}^2_{\mathcal{M}_1} \\ \bar{\mathcal{A}}_{i,i}(t)x(t) & t \in [t_k + \tau, t_{k+1}),\ (i, i) \in \mathcal{I}^2_{\mathcal{M}_2} \end{cases} \tag{5}$$

where $\bar{\mathcal{A}}_{i,j}(t) = A_i + B_i\mathcal{K}_j(t)$, $\bar{\mathcal{A}}_{i,i}(t) = A_i + B_i\mathcal{K}_i(t)$.

Problem 3. Consider switched system (1) with a switching delay τ, design a state feedback control scheme (4) guaranteeing the exponential stability of closed loop (5).

At last when τ is uncertain, Problem 3 can be developed accordingly as follows.

Problem 4. Consider switched system (1) with switching delay, design a state feedback control scheme (4) with maximal ability of tolerating switching delay τ^* guaranteeing the exponential stability of closed loop (5).

The above four linked problems are the main concerns to be addressed in the rest of this paper.