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Self-adjusting leakage type adaptive robust control design for uncertain systems with unknown bound

Rongrong Yu [a,*], Ye-Hwa Chen [b], Han Zhao [a], Kang Huang [a], Shengchao Zhen [a,*]

[a] School of Mechanical Engineering, Hefei University of Technology, Hefei, Anhui 230009, China
[b] The George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA

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ABSTRACT

The control problem for an uncertain mechanical system is addressed. The mechanical system under consideration, which is to follow a class of prescribed constraints, contains (possibly fast) time-varying uncertainty. The uncertainty is bounded, but the bound is not necessarily known. A new class of robust control schemes is proposed, which is based on on-line adaptation of a design parameter. Two categories of adaptation laws, both are based on self-adjusting leakage, are proposed. The control guarantees uniform boundedness and uniform ultimate boundedness of a performance measure. Comparing with the previous control scheme, the importance of this new control is that it can compensate the uncertainty in a very effective way. It also avoids over compensation and renders modest control effort. For demonstration purpose, the Toda Lattice, an ideal model of mechanical system with asymmetrical behaviour is chosen. The energy control of the Toda Lattice demonstrates the validity and reliability of control schemes and adaptation laws.

1. Introduction

For control design of constrained mechanical systems, it is essential to investigate the fundamental properties related to the system and constraint. Obtaining the equation of motion of the constrained mechanical system is one of the central issues. The quest has been active since Lagrange who enunciated the Lagrangian (or analytical) mechanics in 1787.

The constraint problem can be divided into passive constraint and servo constraint [1]. The passive constraint is the most extensively studied subject in Lagrangian mechanics. Many important contributions about passive constraint have been made [2]. In the servo constraint problem, the main focus is to find the servo mechanisms' (such as motors') constraint force/torque, so that constraints are followed. There have been several successful development of constraint control designs, some of which are based on exact model-based designs [1,3,4]. On the other hand, uncertainties in the system are often inevitable in practice. The research on the control of mechanical systems with uncertainties has always been very active. Many efforts dealing with uncertainty mechanical systems have been made [5,6].

Once the control design is completed, a realistic test bed would be needed. In practice, it is usually very difficult to find mechanical system with perfectly linear and symmetrical elements: most elements are nonlinear and asymmetrical. One may find spring's stiffness grows as being stretched or compressed. One may also find spring is stronger in tension and weaker under compression, or the other way around. Others may include flexible cables, which in suspension bridges

* Corresponding authors at: School of Mechanical Engineering, Hefei University of Technology, Hefei, Anhui 230009, China.
E-mail addresses: rongala91@163.com, yurongrong@mail.hfut.edu.cn (R. Yu), yehwa.chen@me.gatech.edu (Y.-H. Chen), hanzhaoff@qq.com (H. Zhao), hfhuang98@163.com (K. Huang), zhenshengchao@qq.com, zhenshengchao@hfut.edu.cn (S. Zhen).

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may be stronger under tensile force and weaker under compressive force. The nonlinear lattice with spring elements is an ideal model to demonstrate all these nonlinearity and asymmetry. It should serve as a suitable test bed for control design. The research of lattice also has wide engineering applications due to the disparate nature under different forces.

The study of nonlinear lattices problem has mystified scientists since Fermi, Pasta and Ulam (FPU) did the seminal work in 1955 [7]. Instead of analysing the response of each mass in the nonlinear lattice, FPU studying the total energy in the lattice. Up to now, the FPU problem is still a research area which is active, and there are many questions related to its significance for science that still unanswered [8]. In 1960s, inspired by the FPU paradox, Toda [9] found analytical solutions to the nonlinear lattice which is composed of exponential spring elements. In this paper, the nonlinear lattice is called "Toda Lattice". Spring elements are stronger in tension and weaker under compression, or vice versa.

Energy control of Toda Lattice has been investigated by Polushin [10] and Udwadia [11]. These controllers all can develop energy very perfect, but they did not address uncertainty. The main focus of this paper, in this regard, is to propose adaptive robust controls and to control the energy of the Toda Lattice with uncertainty to the desired level. The uncertainty considered includes uncertain parameters in the modeling, which are (possibly fast) time-varying and bounded. The bound is unknown. Under the presence of uncertainty and initial condition deviation from the constraints, it is desired to drive the system to follow the designated constraints.

Much effort on adaptive control, robust control, and robust adaptive control has been made [12–19]. The adaptive robust control, a less recognized endeavor, also has been developed [20–26]. Adaptive robust control also has been applied in industrial applications [27–29]. The state of the art of adaptive robust control mainly focuses constant leakage. Leakage is a technique in adaptation. It can be used in robust adaptive control (in which robustness is added to adaptive control design) as well as adaptive robust control (as the current paper, in which adaptation characteristic is added to robust control design). By using the Udwadia-Kalaba equation [30], a new adaptive robust control is proposed and also applied to the energy control of nominal Toda Lattice when uncertainty is involved. Comparing with a previous work in [31], we creatively introduce the self-adjusting leakage which can influence the rate of change of an adaptive parameter based on the system performance. Two adaptation laws, which are both of self-adjusting leakage type but with different characteristics, are proposed. Compared with the adaptive robust control with a constant leakage, the self-adjusting leakage type adaptive robust control can compensate the uncertainty in a very effective way and can also avoid over compensation. Meanwhile, using the control with the self-adjustment leakage will approach the desired goal faster than using the control with a constant leakage.

The main contributions of this paper are fourfold. First, when no uncertainty is present, a nominal control which is based on the analytic expression of constraint force [32–34] is presented. This is in conformity with Gauss's minimum principle and is therefore modest in magnitude. Second, two adaptation laws are constructed to emulate a constant design parameter vector, which maybe relevant (but not necessarily identical) to the uncertainty bound. The self-adjusting leakage mechanism in the adaptation laws is performance dependent, which affects the amount of leakage. The leakage type can compensate the effect of the uncertainty effectively. It also avoids over compensating the system. Third, based on the two adaptation laws, adaptive robust controls are proposed, which are able to guarantee uniform boundedness and uniform ultimate boundedness. Fourth, we demonstrate the control design in the Toda Lattice which is an ideal model of the mechanical system with asymmetrical behaviour. The superiority of both adaptive robust controls is shown in simulations. We also show that the choices of control design parameters are not unique, many choices are available. This flexibility broadens the applicability in many applications.

2. Mechanical system under constraints

Consider a mechanical system which can be represented as follows [35,36]:

$$M(q(t),\sigma(t),t)\ddot{q}(t) + C(q(t),\dot{q}(t),\sigma(t),t)\dot{q}(t) + G(q(t),\sigma(t),t) + F(q(t),\sigma(t),t) = \tau(t). \tag{1}$$

Here $t \in \mathbf{R}$ is the time, $q \in \mathbf{R}^n$ is the coordinate, $\dot{q} \in \mathbf{R}^n$ is the velocity, $\ddot{q} \in \mathbf{R}^n$ is the acceleration, $\sigma \in \mathbf{R}^p$ is an uncertain parameter which is (possibly fast) time-varying, and $\tau \in \mathbf{R}^n$ is the control force. Moreover, $M(q,\sigma,t)$ is the $n \times n$ inertia matrix, $C(q,\dot{q},\sigma,t)\dot{q}$ is the $n \times 1$ Coriolis/centrifugal force, $G(q,\sigma,t)$ is the $n \times 1$ gravitational force, and $F(q,\sigma,t)$ is the $n \times 1$ friction force or other external disturbance. The uncertain parameter $\sigma \in \Sigma \subset \mathbf{R}^p$ where the bounding set Σ is compact and possibly unknown. The functions $M(\cdot), C(\cdot), G(\cdot)$, and $F(\cdot)$ are continuous.

Consider the following constraints

$$\sum_{i=1}^{n} A_{li}(q,t)\dot{q}_i = c_l(q,t), \; l = 1,2,\cdots,m, \tag{2}$$

where $\dot{q}_i$ is the i-th component of $\dot{q}, A_{li}(\cdot)$ and $c_l(\cdot)$ are both C^1 in q and $t, 1 \leqslant m \leqslant n$. These constraints are in the form of *first-order* and may be non-integrable and nonholonomic. They can be written in the matrix form

$$A(q,t)\dot{q} = c(q,t), \tag{3}$$

where $A = [A_{li}]_{m\times n}, c = [c_1 \; c_2 \; \cdots \; c_m]^T$.