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A new bivariate dimension reduction method for efficient structural reliability analysis

Jun Xu [a,b,*], Chao Dang [a]

[a] College of Civil Engineering, Hunan University, Changsha 410082, PR China
[b] Hunan Provincial Key Lab on Damage Diagnosis for Engineering Structures, Hunan University, Changsha 410082, PR China

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A B S T R A C T

This paper presents a new bivariate dimension reduction method (BDRM) for statistical moments evaluation and structural reliability analysis with accuracy and efficiency. A high-order unscented transformation (HUT) is introduced to evaluate the two-dimensional integrals involved in BDRM, and the free parameter involved in HUT is suggested. In this regard, the proposed BDRM can be formulated accordingly for statistical moments assessment. Then, the performance function's probability density function (PDF) is reconstructed by the shifted generalized lognormal distribution (SGLD) with the available statistical moments as constraints. Thus, the failure probability can be straightforwardly obtained by a simple integral over the PDF. The proposed method is verified by five numerical examples, including linear and non-linear, explicit and implicit performance functions. Besides, some other existing methods are also employed to demonstrate the advantages of the proposed method. It is found that the proposed method can keep the trade-off of accuracy and efficiency for structural reliability analysis.

1. Introduction

The primary problem in structural reliability analysis is to evaluate the failure probability, which is essentially a multi-dimensional integral defined by [1,2]

$$P_f = \text{Prob}\,[G(\mathbf{X}) \leqslant 0] = \int_{\Omega_f} p_{\mathbf{X}}(\mathbf{x})\mathrm{d}\mathbf{x} \tag{1}$$

where P_f denotes the failure probability; $\mathbf{X} = [X_1, X_2, \cdots, X_n]^T$ is the vector collecting all n basic random variables involved in structural properties and loading conditions, such as material properties, sectional dimensions and external loads, etc.; $Z = G(\mathbf{X})$ is the performance function or the limited state function; $\Omega_f = \{\mathbf{X}|G(\mathbf{X}) \leqslant 0\}$ denotes the failure domain in the basic random-variate space; and $p_{\mathbf{X}}(\mathbf{x})$ is the joint probability density function (PDF) of $\mathbf{X}$. Although this definition is quite simple, difficulty still arises since the analytical solution of $Z = G(\mathbf{X})$ is usually unavailable except for some special cases due to the high nonlinearity and a large number of random variables involved in practical problems. This difficulty has motivated various approximation techniques proposed and developed for structural reliability assessment over the past several decades. Generally, three categories of methods could be found.

* Corresponding author at: College of Civil Engineering, Hunan University, Changsha 410082, PR China.
E-mail addresses: xujun86@hnu.edu.cn (J. Xu), chaodang@outlook.com (C. Dang).

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Sampling-based simulation methods are widely used for evaluating the failure probability, which can be classified under the first category. It is well-known that the crude Monte Carlo simulation (MCS) [3] can provide accurate results if sufficient number of samples are employed. However, the computational effort could be prohibitively large when the small failure probability (i.e. $P_f \leqslant 10^{-3}$) is of concern and a time-consuming model evaluation is involved. In this regard, it is merely used as a benchmark method to check the accuracy of newly developed methods. Although various improvements such as the Importance Sampling [4–6], Directional Sampling [7,8], Subset Simulation [9], Line Sampling [10,11] and Asymptotic Sampling [12] etc. have been developed to further enhance the efficiency of the crude MCS, these methods may still require considerably large computational time to obtain results with reasonable accuracy.

To decrease the computation burden, another route named surrogate model based methods has attracted increasing attention for structural reliability analysis. They aim at building a simplified explicit mathematical form in place of the original performance function, thus the computational effort could be greatly reduced. Some representative surrogate model based methods, e.g. response surface method [13–15], polynomial chaos expansion[16–18], Kriging methods [19–21], artificial neural networks [22–24], support vector machines [25–27], have been widely applied in structural reliability analysis. Nevertheless, it is usually quite difficult to quantify the error of using a surrogate model to substitute the original performance function [28].

Moment methods can be classified under the third category for structural reliability analysis. Among them, the first-order reliability method (FORM) and second-order reliability method (SORM) [2] are regarded as the most popular methods in the past several decades. The FORM or SORM is based on the first- or second-order Taylor expansions of the performance function at the most probable point (MPP). Thus, a derivative-based iterative search process is required to locate the MPP. Though the FORM or SORM can give reasonably accurate results for some practical problems, numerical difficulties in the MPP search and inaccuracies could be encountered when the nonlinear implicit performance function is considered [29]. The derivative-based moment methods also include the perturbation method [30–33], Neumann expansion [34,35] and so on. On the other hand, the derivative-free moment methods are of great interest for structural reliability analysis, where a finite number of statistical moments are first evaluated and then the reliability can be derived based on these moments. In this regard, the evaluation of statistical moments, which involves multi-dimensional integrations, plays an important role to the accuracy and efficiency for reliability considerations. Although, several methods, e.g., the tensor product method [36], sparse grid method [37], cubature formulation [38,39], and quasi-symmetric point method [40–42] have been developed for this propose, the method which can keep the trade-off of accuracy and efficiency for moments evaluation, is still of great interest.

The dimension reduction method (DRM) [43,44] is another attractive method for moments evaluation, since it transforms a multiple-dimensional integration to be several low-dimensional integrations for moments evaluation. Therefore, the difficulty of computing a multi-dimensional integration can be avoided to some extent. The present study is specifically interested in the DRM. The univariate dimension method (UDRM) [43] and the eigenvector UDRM [45] could be the most popular ones, which require considerably few performance function calls since only several one-dimensional integrations are involved. However, for the high-order moments evaluation of strongly nonlinear performance functions, it is found that those UDRMs may be inadequate due to two- and other-higher dimensional integrations contained in the residual error [38,44]. Alternatively, a multiplicative-univariate dimensional reduction method (M-UDRM) is proposed in [46] for fractional raw moments estimation, which is proved to be quite efficient and accurate for moderate nonlinear performance functions. Nevertheless, it still may not be adequate for the high-order central moments evaluation of a highly nonlinear performance function since this method is UDRM in essence. To improve the accuracy, the bivariate dimension reduction method (BDRM) can be employed [44,47,48], which only includes several two-dimensional and one-dimensional integrals. It is known that, some transformations, e.g., the Rosenblatt or Nataf transformation, can be used to transform the original random variables into independent standard normal variables. In this regard, only Gaussian-weighted integrals need to be considered. In literatures, the Gauss-Hermite integration (GHI) scheme is widely adopted to deal with the one- and two-dimensional integrals involved in BDRM [44,49–51]. However, a large number of points are actually required in BDRM to achieve a satisfactory accuracy. For example, if the five-point GHI scheme is applied, a total of 25 points are involved in one two-dimensional integral due to the tensor product rule, which may result in computationally expensive efforts for a performance function including multiple variables. Hereafter, this BDRM is referred to as the original BDRM. In contrast, the three-point GHI scheme, which is denoted as GHI2, may be efficient since only 9 points are needed in one two-dimensional integral. Unfortunately, it may lead to much larger errors than the five-point GHI scheme for the two-dimensional integrals involved in BDRM. In this regard, the improvements on the efficiency of the original BDRM or the accuracy of the BDRM with GHI2 could be of great necessity. In the present paper, a new BDRM, which can keep the tradeoff of accuracy and efficiency, is established based on employing a high-order unscented transformation (HUT) scheme [52] to evaluate the two-dimensional integrals involved in BDRM. It will be found that the proposed method is as efficient as the BDRM with GHI2, however, it is much more accurate and robust than that. Besides, compared with the original BDRM, the proposed method is able to capture almost the same accuracy for moments estimation with much fewer deterministic calculations.

Once the first-four moments of the performance function are calculated by the proposed BDRM, a versatile distribution model named the shifted generalized lognormal distribution (SGLD) [53], which has a rich flexibility in shape and nearly encompasses the entire skewness-kurtosis permissible for unimodal densities, is fitted to recover the performance function's PDF. Then, the failure probability can be obtained by a simple integral over the PDF. The rest of the paper is organized as