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# A nonlocal strain gradient theory for dynamic modeling of a rotary thermo piezo electrically actuated nano FG circular plate

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ABSTRACT

In this paper, vibrational behavior of functionally graded piezoelectric material (FGPM) in a nano circular plate subjected to rotational and thermal loads is studied. Thermo-electro–elastic characteristics of FGPM nano circular plate are changed continuously along the thickness with the power-law model. This analysis is based on nonlocal strain gradient theory (NSGT), strain gradient theory (SGT) and nonlocal Eringen theory (NET) within the first order shear deformation theory (FSDT). Via Hamilton's principle, the governing equations of FGPM nano circular plate obtained and solved using the differential quadrature method (DQM). It is seen that the vibrational behavior of FGPM nano circular plate is considerably affected by temperature rises, angular velocity, external voltage, FG power index, nonlocal parameter and material length scale for clamped and hinged boundary conditions.


## 1. Introduction

Functionally graded materials (FGMs) are a particular branch of composites that have spatial variations in the volumetric portion of their phases. These variations give us more possibilities in designing materials [1]. The static and dynamic behaviors of FGMs beams [2], in shells and plates have been studied by researchers [3–7]. Lately, FGMs have been used in structures such as micro-electro-mechanical systems (MEMS) for achieving high sensitivity and acceptable performance [8–12], also these materials are a combination of two or more different materials, and their physical properties are continually different from their dimensions, which has led to the use in various engineering applications, including marine and aerospace [13–17]. A lot of studies in recent years have been devoted to the vibration analysis and studying mechanical behavior of materials in plates [18–20], shells and carbon nanotube-reinforced composites [21–23] and panels [24–28]. Recently, a large number of articles and researches efforts on circular vibrations which have been developed using various theories [23] and methods of the plates. Aghelinejad et al. [29] used a shooting method to analyze the nonlinear bending and simulate the thermal bending of FGM thin circular plates with boundary conditions. Alipour and Shariyat [30] examined double superposition zigzag theory and used the principle of minimum energy potential for studying and analyzing the stress and deformation of sandwich plates subjected to non-uniform shear tractions. Allahverdizadeh et al. [31–33] investigated the nonlinear behavior of a thin-film circular plat [34,35], which was investigated under thermal conditions with clamped boundary conditions. Ebrahimi et al. [36,37] examined the vibrational motion of circular plates using differential equations. Hassan and

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Mehran [34] applied three-dimensional linear elements and the energy method on the static and dynamic behavior of a relatively thick sheet. The plate was made of two dimensional graded materials while, three-dimensional elastic theory is also used in their research. According to the study of the above articles, the accuracy of calculations and boundary conditions, is strongly dependent on computational methods and the choice of theories of the plate [8,38–40]. Choosing the proper theory on the plate is the first step and the cornerstone of the whole process of computing and analyzing the problem. The plate theories have different types, including classical plate theory (CPT) [8,11,36], higher-order shear deformation plate theory (HSDT) [8,41], three-dimensional (3D) elasticity theory and first-order shear deformation plate theory (FSDT) [11,12,42,43] It is known that, classical plate theory (CPT) is the simplest theory in this field that overlooks shape and its changes in the direction of thickness. This theory gives accurate results for thin plates, but if the plate is relatively thick or when the exact solution for higher vibration modes is desired, the results may be inaccurate. To eliminate the defects of classical plate theory (CPT), first-order shear deformation plate theory (FSDT) was developed that made the shear modification factor in terms of natural deformation through the thickness of the plate. The value of the shear modification factor is considered to change depending on boundary conditions, material properties and loading conditions. Higher-order shear deformation plate theory (HSDT) are based on the assumption of high-order changes in the displacement inside the plate-thickness. Three-dimensional (3-D) elasticity theory does not rely on these assumptions and can solve the problem of vibration of thick plates with any numerical accuracy, but, like higher-order shear deformation plate theory (HSDT), it needs more computing resources to get accurate results. In order to obtain the accurate results of the vibration, the thickness of the circular plate theory is used. Based on modern theory, computational methods play an important role in understanding FGM-behaviors. Reviewing articles, show that the commonly used methods are: differential equations method, finite element method, Galerkin's method [44–47], discrete singular convolution method [48–51] exact solution and so on. According to the studies, it seems that the method is applicable only to a particular type of classical boundary conditions that is simply-supported supports. Boundary conditions of circular plates of FGM are not always same and includes wide range of variations, such as elastic constraints. Therefore, we need to find an integrated and usable method that is capable to solve more complex problems. A number of new results from circular planes and their operation with elastic boundary conditions are presented which can be the next basic solution to computational methods. In addition, the effects of boundary conditions and material parameters and geometric conditions on the vibration of the plate and its performance have also been investigated. In recent years, due to the special properties and superiority of Nano-structures, they have focused on several directions. Experimental results show that when the scales go to the nano or micro scale, atomic forces play an important role. So, if we do not consider these effects, our solution may be incorrect. Eringen's nonlocal elasticity theory [52] is an important one, which is one of the well-known theories for the theory of continuous environment mechanics that considers the effect of small scale with great care.

This paper present the study the effects of parameters like, nonlocal parameter ($\mu_{NSGT}$), length scale parameter ($l_{NSGT}$), FG power index ($n$), temperature ($\Delta$T), angular velocity and external voltage for Hinged and Clamped boundary conditions on the non-dimensional first natural frequency of nano circular plate based on nonlocal strain gradient theory (NSGT), strain gradient theory (SGT) and nonlocal Eringen theory (NET).

## 2. Formulation

### 2.1. Preliminaries

The considered functionally graded electro elastic (FGPM) nano circular plate with the thickness of h and the radius of R is schematically shown in Fig. 1. Also, the thermo-electro-mechanical properties of the nano circular plate are presented in Table 1.

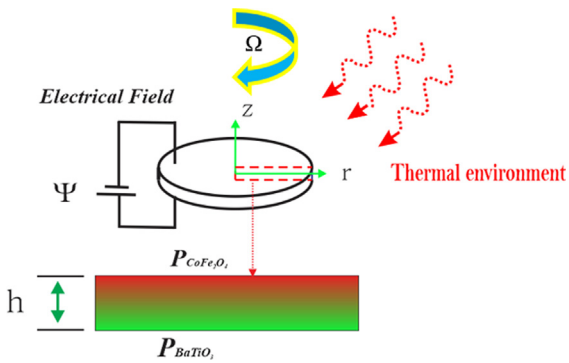


**Fig. 1.** The schematic view of the rotating functionally graded electro elastic (FGPM) nano circular plate.