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A signal processing framework for operational modal analysis in time and frequency domain

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ABSTRACT

In operational modal analysis (OMA), correlation functions, sometimes referred to as covariance functions, are commonly used for modal parameter extraction. Other techniques for parameter estimation use spectral density estimates. There are several known techniques for computing spectral density and correlation functions. The most common technique for spectral density estimates, is Welch's method. A more infrequently used technique, however, is based on computing the discrete Fourier transform (DFT) of the entire signals, and multiplying these DFTs into auto and cross-periodograms. To produce a correlation function, the inverse Fourier transform of the periodogram is used. To produce spectral density estimates, the periodogram may be smoothed. In the present paper this method of computing the spectral and correlation functions is investigated, and compared to other methods of spectral and correlation estimation. It is shown that the method has several advantages not only for estimation of spectra and correlation functions, but also because filtering, integration and differentiation, removal of harmonics, and compensation for non-ideal sensor characteristics are functions that can readily be encompassed in this technique, with high performance, at a minimum of computational cost. Furthermore, two methods to remove harmonics in spectral densities as well as in correlation functions, are developed in the paper. The first method, frequency domain editing (FDE), removes one or more stable harmonics, where variations of the frequency are small. The other method, order domain deletion (ODD), works in cases where the frequency of the harmonic, or harmonics, varies, and where the instantaneous frequency as a function of time is known, for example by processing a tacho signal. Based on the several advantages with using long DFTs as the estimation method for spectra and correlation functions, it is recommended as the standard framework for signal processing in OMA applications.

1. Introduction

Operational modal analysis (OMA) was originally developed using free decay responses on large scale civil engineering structures [1–3]. Later, correlation functions were used [4]. OMA parameter estimation algorithms using correlation functions have some advantage over other methods, such as data-driven stochastic subspace methods, in that the correlation functions can be plotted, which may give engineering insights into the properties of the system. In recent years, frequency domain methods have also gained some popularity, [5,6].

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How to estimate correlation functions has been known for a long time. Many mathematical works, such as [7], for example, include a discussion on how to estimate correlation functions. In the vibration analysis community, the early work [8] and later [9,10] are well-known standard texts. Common for all those books is that they recommend using Welch's method for averaging discrete Fourier transform (DFT) results, and finally taking the inverse DFT of the result to produce the correlation function estimate.

For spectral estimates, Welch's method [11] is also the most commonly used method, although it is implemented differently than when used for correlation estimates, as a time window such as the Hanning window must be applied to control leakage in the former case. In cases where both spectra and correlation function estimates are desired, there is thus no synergy, but each function has to be estimated separately.

Presently, OMA is used increasingly outside its original applications in civil engineering. In some current applications, harmonics are therefore present in the response signals stemming from, for example, rotating components in a wind turbine, or propeller induced vibrations on a ship. Harmonics are well known to pose a problem for operational modal analysis, since harmonics essentially correspond to poles with zero damping. There is therefore a need for methods to deal with response data that contain harmonics. The problem of harmonics in data for operational modal analysis has therefore been addressed by many studies in the past [12–25]. In general, methods to treat harmonics in response signals either try to include the harmonics in the fitted model, or to remove the harmonics prior to estimating the system identification functions; in most cases cross-correlation functions.

In [26] a method was proposed to be used for removing (relatively stable) harmonics in spectrum estimates, although without any analysis. The proposed method essentially relies on using a smoothed periodogram for the estimation of spectral density. The harmonics are removed from the periodogram prior to smoothing it, producing a spectral density estimate without the harmonics. In the present paper we name this method the *frequency domain editing* (FDE) method. Another novel method, the *order domain deletion* (ODD) method, is then developed to remove harmonics in cases with large frequency variation. The ODD method relies on resampling the signal using a known rpm-time profile, which could be estimated from a tacho signal, for example.

In the present paper, the periodogram method is also further developed to efficiently estimate correlation functions, with the aim of developing a signal processing framework which allows an efficient procedure to estimate correlation functions as well as spectral density estimates for OMA applications. The framework should, in addition to being able to remove harmonics, also offer the capability of efficient signal processing such as integration and differentiation and lowpass and highpass filtering etc., and the capability of efficiently producing both correlation functions and spectral densities. Furthermore, the method should also be robust to real-life data. If all these features are included, the method will be versatile in that it supports parameter estimation techniques that are either based on correlation functions (also called covariance-driven), and frequency domain methods based on spectra. This means the method will work together with, for example, Ibrahim time domain (ITD) and covariance-driven SSI in time domain, or the frequency domain decomposition (FDD) and the least squares complex frequency domain method (LSCF) in the frequency domain.

The paper is organized as follows: In Section 2, a theoretical presentation of correlation and spectral estimation is presented. A new framework that allows to estimate both correlation functions and spectral densities is developed. The framework is shown to allow efficient frequency domain signal processing for filtering, integration and differentiation, and correction of non-ideal sensor characteristics. In Section 3, two methods for removing harmonics in estimates of correlation and spectral estimates are developed. In Section 4 we present simulation results that demonstrate that the proposed method can be used to efficiently estimate correlation functions in cases where there are harmonics in the original measured response signals. In Section 5 we investigate the performance of the methods for removing harmonics. In Section 6, finally, the methods and results are summarized.

2. Correlation and spectral estimation theory

2.1. Correlation functions

The cross-correlation function of two (weakly) stationary and ergodic random signals, $x(t)$ and $y(t)$ may be defined by

$$R_{yx}(\tau) = \mathrm{E}[y(t)x(t-\tau)] \tag{1}$$

where E is the expected value operator, and τ the time lag in seconds. The autocorrelation function $R_{xx}(\tau)$ can be defined by the same equation, by setting $y(t) = x(t)$. To estimate the correlation function in the time domain, the expected value operator is replaced by a mean value. Since we are interested in real-life applications we also introduce sampled signals $x(n)$ and $y(n)$, and use m to denote the discrete time lag. An estimator for the discrete case then becomes

$$\widehat{R}^{b}_{yx}(m) = \frac{1}{N}\sum_{n=0}^{N-1}(y(n)x(n-m)). \tag{2}$$

It is easy to realize that, assuming the lengths of $x(n)$ and $y(n)$ are N samples, then for all but the zero time lag, $m = 0$, there will not be N overlapping values between $y(n)$ and the time translated $x(n-m)$, but rather $N-m$ samples, for positive lags m. This means that the estimator in Eq. (2) is a *biased estimator* as is very well known. It is, however, easy to define an