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Brief paper

Robust finite-time output feedback control of perturbed double integrator[☆]

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ABSTRACT

This paper addresses the problem of finite-time output feedback stabilization for the perturbed double integrator system. A simple output feedback proportional–derivative (PD) controller is proposed. Global finite-time stability is proven based on Lyapunov stability theory and geometric homogeneity technique. Furthermore, it is proven that the proposed controller can maintain local finite-time stability regardless of some nonlinear perturbations. Thus, the proposed controller actually can be applied to a large class of uncertain second-order nonlinear systems. Simulations demonstrate the effectiveness of the proposed approach.

1. Introduction

Finite-time stabilization of dynamical systems may give rise to fast transient and high precision and robustness performances besides finite-time convergence to the equilibrium (Bhat & Bernstein, 1998, 2005; Haimo, 1986). Finite-time stabilization of systems is quickly developing in the last decades, and several schemes can be found in the literature (Amato, Ambrosino, Cosentino, & De Tomais, 2010; Bhat & Bernstein, 1998, 2005; Hong, 2002; Hong & Jiang, 2006; Huang, Lin, & Yang, 2005; Levant, 2001, 2005, 2007; Moulay & Perruquetti, 2008; Nagesh & Edwards, 2014; Orlov, 2005; Shtessel, Taleb, & Plestan, 2012; Zhang, Feng, & Sun, 2012).

A main drawback for these finite-time controls is that the full state information is assumed to be available for feedback. To overcome this restriction, several output feedback controls for finite-time stabilization of double integrator have been proposed. Specifically, Hong, Huang, and Xu (2001) solve the problem of output feedback finite-time stabilization for the double integrator and extend to a large class of second-order systems, based on finite-time separation principle. Orlov, Aoustin, and Chevallereau (2011) show that a modification of the twisting controller and the supertwisting observer can be coupled together to present a unified framework for the output feedback finite-time stabilization of a perturbed double integrator. Bernuau, Perruquetti, Efimov, and Moulay (2012) combine a homogeneous observer with a homogeneous control to ensure global finite-time stabilization of the double integrator systems. Other output feedback finite-time stabilizing controls for dynamical systems can be found in Amato, Ariola, and Cosentino (2006), Angulo, Fridman, and Levant (2012), Angulo, Fridman, and Moreno (2013), Dinuzzo and Ferrara (2009), Li and Qian (2006), Li, Qian, and Ding (2010), Plestan, Moulay, Glumineau, and Cheviron (2010) and Qian and Li (2005).

This paper presents an alternative design for output feedback finite-time stabilization of perturbed double integrator. A very simple but effective output feedback nonlinear PD controller is proposed. A simple nonlinear filter is constructed to replace the velocity measurement. The proposed filter does not refer to the control input. The benefit of this design is that the controller can be designed separately from the filter and provides much flexibility for the control gains selection with an improved performance. Lyapunov stability theory and geometric homogeneity are employed to prove global finite-time stability. Using homogeneous techniques presented in Hong et al. (2001), it is also shown that the proposed controller can retain local finite-time stabilization of second-order systems with a class of nonlinear perturbations. This robust property actually extends the applicability of the proposed

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controller to a large class of uncertain second-order nonlinear systems. Simulations are presented to verify the effectiveness of the proposed approach.

2. Preliminaries

Some concepts of finite-time stability of nonlinear systems are reviewed following the approach of Bhat and Bernstein (1998, 2005) and Hong et al. (2001).

Definition 1 (*Finite-Time Stability, FTS*). Consider the system

$$\dot{x} = f(x), \quad f(0) = 0, \ x \in \Re^n \tag{1}$$

where $f : U_0 \to \Re^n$ is continuous on an open neighborhood U_0 of the origin. Suppose that system (1) possesses unique solutions in forward time for all initial conditions. The equilibrium $x = 0$ of system (1) is (locally) finite-time stable if it is Lyapunov stable and finite-time convergent in a neighborhood $U \subset U_0$ of the origin. The finite-time convergence means the existence of a function $T : U \setminus \{0\} \to (0, \infty)$, such that, $\forall x_0 \in U \subset \Re^n$, the solution of (1) denoted by $s_t(x_0)$ with x_0 as the initial condition is defined and $s_t(x_0) \in U \setminus \{0\}$ for $t \in [0, T(x_0))$, and $\lim_{t \to T(x_0)} s_t(x_0) = 0$ and $s_t(x_0) = 0$ for $t > T(x_0)$. When $U = \Re^n$, we obtain the global finite-time stability (GFTS).

Definition 2 (*Homogeneity*). A function $V : \Re^n \to \Re$ is homogeneous of degree d with respect to (w.r.t.) weights $r = (r_1, \ldots, r_n) \in \Re^n_+$, if for any given $\varepsilon > 0$, $V(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n) = \varepsilon^d V(x)$, $\forall x \in \Re^n$. A vector field f is homogeneous of degree d w.r.t. weights r, if for all $1 \le i \le n$, the ith component f_i is a homogeneous function of degree $r_i + d$. The system (1) is homogeneous of degree d if f is homogeneous of degree d.

The following results give a sufficient condition for FTS of nonlinear systems.

Lemma 1 (*Bhat & Bernstein, 2005*). *Suppose that system (1) is homogeneous of degree d. Then the origin of the system is finite-time stable if the origin is asymptotically stable and $d < 0$.*

Lemma 2 (*Hong et al., 2001*). *Consider the following system*

$$\dot{x} = f(x) + \hat{f}(x), \quad x \in \Re^n \tag{2}$$

where $f(x)$ is a continuous homogeneous vector field of degree $d < 0$ w.r.t. $(r_1, \ldots, r_n)$ satisfying $f(0) = 0$, and $\hat{f}(x)$ is also a continuous vector field satisfying $\hat{f}(0) = 0$. Assume that $x = 0$ is an asymptotically stable equilibrium of the system $\dot{x} = f(x)$. Then $x = 0$ is a locally finite-time stable equilibrium of the system (2) if

$$\lim_{\varepsilon \to 0} \frac{\hat{f}_i(\varepsilon^{r_1} x_1, \ldots, \varepsilon^{r_n} x_n)}{\varepsilon^{d+r_i}} = 0, \quad i = 1, \ldots, n, \ \forall x \neq 0 \tag{3}$$

uniformly for any $x \in S^{n-1}$.

3. Main results

In this section, output feedback controller for global finite-time stabilization of double integrator is first proposed. Then it is extended to perturbed double integrator systems.

3.1. A solution for double integrator

Consider the following double integrator system

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = u \end{cases} \tag{4}$$

where x_1 and x_2 are the states of the system, u is the input. Our first objective is to design a simple output feedback nonlinear PD controller such that the zero solution of the closed-loop system composed of (4) and the control law is global finite-time stable. The following definition is used

$$\mathrm{sig}^{\alpha}(z) := |z|^{\alpha} \,\mathrm{sgn}(z) \tag{5}$$

where $z \in \Re$, $\alpha > 0$, and $\mathrm{sgn}(\cdot)$ is the standard signum function.

We propose the following output feedback finite-time PD (OFPD) controller to solve the above stated problem:

$$u = -k_p \mathrm{sig}^{\alpha_1}(x_1) - k_d \mathrm{sig}^{\alpha_1}(\upsilon) \tag{6}$$

$$\dot{q}_c = -a \mathrm{sig}^{\alpha_2}(q_c + b x_1) \tag{7}$$

$$\upsilon = q_c + b x_1 \tag{8}$$

where k_p, k_d, a, b, and $0 < \alpha_1 < 1$ are positive constant design gains, $\alpha_2 = (\alpha_1 + 1)/2$, and q_c is an auxiliary variable.

Taking the time derivative of (8) and after substituting (7) into the expression and using (4), yields

$$\dot{\upsilon} = -a \mathrm{sig}^{\alpha_2}(\upsilon) + b x_2. \tag{9}$$

Substituting the control input (6) into (4), it follows that

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -k_p \mathrm{sig}^{\alpha_1}(x_1) - k_d \mathrm{sig}^{\alpha_1}(\upsilon). \end{cases} \tag{10}$$

The main result of this section is given in Theorem 1.

Theorem 1. *The zero solution of the closed-loop system (9) and (10) is globally finite-time stable.*

Proof. The proof proceeds in the following two steps. First, the global asymptotic stability (GAS) is proven following Lyapunov's direct method and LaSalle's invariance theorem. Second, the GFTS is shown using Lemma 1.

Step 1 (GAS analysis). To this end, the positive-definite Lyapunov function candidate is proposed as follows:

$$V = \frac{k_p}{\alpha_1 + 1} |x_1|^{\alpha_1 + 1} + \frac{1}{2} x_2^2 + \frac{k_d b^{-1}}{\alpha_1 + 1} |\upsilon|^{\alpha_1 + 1}. \tag{11}$$

Taking the time derivative of (11) along the closed-loop system (9) and (10), it follows that

$$\dot{V} = k_p \mathrm{sig}^{\alpha_1}(x_1) \dot{x}_1 + x_2 \dot{x}_2 + k_d b^{-1} \mathrm{sig}^{\alpha_1}(\upsilon) \dot{\upsilon}. \tag{12}$$

Upon substituting (9) and (10) into (12), we have

$$\dot{V} = -k_d a b^{-1} \mathrm{sig}^{\alpha_1}(\upsilon) \mathrm{sig}^{\alpha_2}(\upsilon) = -k_d a b^{-1} |\upsilon|^{\alpha_1 + \alpha_2}. \tag{13}$$

Hence, V is a positive-definite Lyapunov function whose time derivative $\dot{V}$ is negative semi-definite. In fact, $\dot{V} = 0$ means $\upsilon = 0$. From (10) and (9), respectively, we have $x_1 = 0$ and $x_2 = 0$. By LaSalle's invariance theorem (Slotine & Li, 1991), we have $x_1(t) \to 0$, $x_2(t) \to 0$, and $\upsilon(t) \to 0$ as $t \to \infty$ for any initial state $(x_1(0), x_2(0), \upsilon(0))$. The GAS of closed-loop system (9) and (10) is completed.

Step 2 (GFTS analysis). The GFTS is proven using Lemma 1. To do so, let $z_1 = x_1$, $z_2 = \dot{x}_1 = x_2$, $z_3 = \upsilon$, and $z = (z_1, z_2, z_3)^T$. The state equation of the closed-loop system (9) and (10) can be rewritten as

$$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = -k_p \mathrm{sig}^{\alpha_1}(z_1) - k_d \mathrm{sig}^{\alpha_1}(z_3) \\ \dot{z}_3 = -a \mathrm{sig}^{\alpha_2}(z_3) + b z_2. \end{cases} \tag{14}$$

Clearly, $z = 0$ is the equilibrium of (14). Moreover, system (14) is homogeneous of degree $d = \alpha_2 - 1 < 0$ w.r.t. (r_1, r_2, r_3) with $r_1 = r_3 = 1$ and $r_2 = \alpha_2$. In addition, it is clear from step 1 that $z = 0$ is the asymptotic equilibrium of (14). Hence, by Lemma 1, the GFTS directly follows. This completes the proof. □