Mechanical Systems and Signal Processing 116 (2019) 997–1016

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Review

An Arnoldi reduction strategy applied to the semi-analytical finite element method to model railway track vibrations

R. Cettour-Janet [a,b,*], A. Barbarulo [a], F. Letourneaux [b], G. Puel [a]

[a] *Laboratoire MSSMat (CentraleSupélec/CNRS UMR 8579/Paris-Saclay University), 3 rue Joliot Curie, 91190 Gif-sur-Yvette, France*
[b] *SYSTRA, 72 rue Henry Farman, 75015 Paris, France*

ARTICLE INFO

Article history:
Received 14 April 2018
Received in revised form 5 July 2018
Accepted 9 July 2018

Keywords:
Railway track
Vibration
Semi-analytical finite element method
Second-order Arnoldi reduction

ABSTRACT

Modelling the vibrational response of railway tracks is a mandatory step to predict rolling noise. This article proposes a model based on the semi-analytical finite element method (SAFE) taking into account the three main characteristics of railway tracks: infinite length in the longitudinal direction, deformable cross-section, and periodic track support. Although this approach is ideally suited to predict rail vibrations, numerical instabilities undermine its reliability at low frequencies. Consequently, a new approach using the second-order Arnoldi reduction (SOAR) is used, which allows to provide the required numerical stability. Lastly, numerical results are compared with classical railway track behaviours described in the literature, as well as experimental data and 3D finite element simulations.

Contents

---

[*] Corresponding author at: Laboratoire MSSMat (CentraleSupélec/CNRS UMR 8579/Paris-Saclay University), 3 rue Joliot Curie, 91190 Gif-sur-Yvette, France.

E-mail addresses: raphael.cettour-janet@centralesupelec.fr, racettour@gmail.com (R. Cettour-Janet).

https://doi.org/10.1016/j.ymssp.2018.07.013
0888-3270/

Nomenclature

List of symbols

$\boldsymbol{\alpha}$	diagonal matrix composed of α_i terms
α_j/β_j	free wave amplitude associate with k_j in the negative/positive direction
$\boldsymbol{\beta}$	diagonal matrix composed of β_i terms
$\boldsymbol{\Delta}(z)/\boldsymbol{\Delta}_z$	diagonal matrix composed of $e^{k_j z}$ terms
δ_{km}	Kronecker symbol
$\boldsymbol{\epsilon}/\boldsymbol{\zeta}$	matrices expressing the linear combinations of free waves to obtain the characteristic waves
$\boldsymbol{\theta}$	sleeper's rotation
λ	propagation constants linked to the periodicity factor of the Floquet theorem
λ_r, μ_r	Lamé parameters
μ	scalar defined as k^{-1} used for the SOAR reduction
$\boldsymbol{\Pi}(z)$	sleeper force projection on the cross-section
ρ	mass density
$\boldsymbol{\phi}_j(x,y)$	jth mode shape of the rail cross-section
$\boldsymbol{\phi}^{FE}$	nodal displacements of $\boldsymbol{\phi}_j(x,y)$
$\boldsymbol{\varphi}$	matrix whose columns are $\boldsymbol{\varphi}_i$
$\boldsymbol{\varphi}_j/\boldsymbol{\psi}_j$	cross section deformation associated with k_j in the negative/positive direction
$\boldsymbol{\psi}$	matrix whose columns are $\boldsymbol{\psi}_i$
ω	angular frequency
$\mathbf{A}$	finite element assembly symbol
A_i	area of the ith element
$\mathbf{A}_p(z)$	matrix expressing the support force as a function of the rail displacement
$\mathbf{B}_0/\mathbf{B}_1/\mathbf{B}_2/\mathbf{B}_3$	finite element stiffness matrices
$\boldsymbol{C}^b$	rotation dynamic stiffness matrix of the ballast
$\mathbf{F}_r$	force vector applied on the rail
$\boldsymbol{F}_{\text{sleeper}}(z)$	force generated by a sleeper
f	frequency
f_{red}	chosen frequency for the creation of the reduction basis
$\boldsymbol{G}(z)$	force vector of the neighbouring generic element
$\mathbf{G}_p(z)$	force vector projection on the rail cross-section
$\boldsymbol{I}$	identity matrix
$\boldsymbol{J}$	sleeper's second moment of area
$\boldsymbol{K}$	diagonal matrix composed of the k_j terms
$\boldsymbol{K}^b$	translation dynamic stiffness matrix of the ballast
$\boldsymbol{K}^p$	translation dynamic stiffness matrix of the pad
$\mathbf{K}_0/\mathbf{K}_1/\mathbf{K}_2$	matrices associated with the QEP of Eq. (3)
k_j/k	complex propagation coefficient
L	distance between two successive sleepers
L_1	distance between the left end of the generic element and the point where the force is applied
M	sleeper's mass
$\mathbf{M}$	finite element mass matrix
$\boldsymbol{N}_i(x,y)$	shape function matrix
N_m	number of modes chosen for the reduction basis
N_n	number of nodes located on the rail foot
n	number of nodes for the cross-section
$\boldsymbol{P}_e$	curl operator
$q_j(z,t)$	wave propagation associated to $\boldsymbol{\phi}_j(x,y)$
$\boldsymbol{R}$	receptance defined as the ratio between the displacement and the force
$\boldsymbol{T\Lambda}$	transfer matrix
t	time
$\boldsymbol{U}$	sleeper's displacement
$\boldsymbol{U}_D$	lines of the reduced basis corresponding to the DoFs of the response
$\boldsymbol{U}_F$	lines of the reduced basis corresponding to the DoFs of the force
$\boldsymbol{U}_e^r$	displacement vector associated with the contact points on the rail
$\boldsymbol{U}_e^s$	displacement vector associated with the contact points on the sleeper
$\boldsymbol{u}(x,y,z,t)$	rail displacement field
$\tilde{\boldsymbol{u}}$	projection of the displacement field of the rail on the reduction basis
$\mathbf{u}_0$	initial vector of the Second order Arnoldi reduction procedure
(x,y,z)	Cartesian coordinates
$\mathscr{G}_N$	second order Krylov subspace