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Statistical moment analysis of nonlinear rotor system with multi uncertain variables

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ABSTRACT

The statistical moments of the dynamical system models with uncertainties are analyzed in this paper. The analytical and numerical cases of the polynomial dimensional decomposition to solve the dynamical responses are provided. First, the polynomial dimensional decomposition method is applied to study a two-degree-of-freedom spring system model with stiffness uncertainty. Second, a linear rotor system model with eight random variables is discussed in the cases of different polynomial orders. Third, a rotor system model supported by cubically nonlinear stiffness is established by the Newton's second law. The amplitude-frequency responses of nine and twelve uncertain variables are calculated by combining the harmonic balance method and the polynomial dimensional decomposition method. The accuracy of the polynomial dimensional decomposition method is verified via comparing with the Monte Carlo Simulation method. The applications of the polynomial dimensional decomposition method in the nonlinear rotor systems can provide theoretical guidance to study complex rotor-bearing systems in the future.

1. Introduction

Study on the engineering system with uncertainty has become one central issue of concerns in actual engineering, attracting the attention of researchers in many areas [1–3]. Batou et al. [4] deals with the identification of a stochastic computational model using experimental eigenfrequencies and mode shapes. Capiez and Soize [5–7] focused on the study of robust design optimization in computational mechanics, structural dynamics and vibroacoustic model. A new approach for the dimensional reduction via projection of nonlinear computational models based on the concept of local reduced-order bases is presented in Ref. [8], and hyper-reduction is applied for design optimization in Ref. [9]. Cataldo et al. [10] analyzed the uncertainty quantification in a voice production mechanical model and update the probability density function corresponding to the tension parameter using the Bayes method and experimental data. Das and Ghanem et al. [11] proposed two numerical techniques to construct a polynomial chaos representation of an arbitrary second order random vector. The predictability of probabilistic model of the human cortical bone with alterations in ultrasonic range is improved by taking into account uncertations in Ref. [12].

In the recent ten years, many researchers concentrated on the study of rotor dynamics with uncertainties. Sinou and Jacquelin [13] developed a stochastic harmonic balance (HB) method with a recursive procedure to evaluate the steady-state response of a rotor system with uncertain stiffness and asymmetric coupling that involves time-dependent terms.

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The dynamic response of a rotor system under uncertain parameters was analyzed by polynomial chaos expansion (PCE) method and the efficiency of the PCE method was verified via comparing with the Monte Carlo Simulation (MCS) method [14]. On the basis of the polynomial chaos, two methods were proposed to carry out the stochastic study of a self-excited non-linear system with friction which is commonly used to represent brake-squeal phenomenon [15]. In Ref. [16], a multi-dimensional HB method coupled with a PCE was used to determine the dynamic response of quasi-periodic dynamic systems subjected to multiple excitations and uncertainties. Recently, a variety of researchers focused on the study of the non-linear dynamic response of a rotor system with common faults and uncertainties [17–19].

Most of the rotor system models are complex and high-dimensional dynamic systems, which usually couples with uncertainty. One of the most commonly used statistical approaches to calculate the response is the MCS method, which includes uncertainties in the deterministic problem [2]. One of the major drawbacks of this approach lies in the computational quantity and a huge number of samples are needed to obtain the statistical response of the system. Many order reduction methods have been proposed to quantify uncertainties of actual engineering so that to solve the computational problems. The common uncertain quantification methods include the PCE method [20–22], the polynomial dimensional decomposition (PDD) method [23–26], etc. A variety of theory derivations and numerical results verified that the PDD method was more efficient and the calculating time is less expensive than the PCE method [27]. The PCE method can commit a larger error than the PDD method for identical expansion orders when the cooperative efforts of input variables on an eigenvalue attenuate rapidly or vanish altogether [28]. So the PDD method is chosen as an uncertain quantification method to solve the corresponding problems in this manuscript. The PDD method has never been used to solve the nonlinear rotor dynamics problems with uncertainty.

The motivation of this paper is to generalize the PDD method to the dynamical system models, excluding the spring, linear rotor and nonlinear rotor system models. In Section 2, the theory of the PDD method is introduced and it is used to study a two-degree-of-freedom (DOF) spring system model with uncertain stiffness. A six-DOF linear rotor system model with nine random variables is applied to verify the accuracy of the PDD method in comparison to the MCS method in Section 3. In Section 4, uncertain cubically linear stiffness is considered based on the linear rotor system model and the PDD method is used to solve the amplitude-frequency response of the nonlinear rotor system, and effects of uncertain parameters are also discussed. Finally, conclusions and outlooks are drawn in Section 5.

2. Introduction to the PDD method in dynamical system

A brief introduction to the PDD method is discussed in this section. First, the analytical case of the steady-state response and the formulas of the univariate PDD method are provided in Section 2.1. The numerical case of the PDD method is also provided via combining with the HB method and the formulas of the bivariate PDD method are listed in Section 2.2.

2.1. The analytical case

A dynamical system can be described by the $n \times n$ mass, damping, and stiffness matrices, $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$, where n is the DOF number. The external excitation force acting on this system can be described by $\mathbf{F}(t)$, and $\mathbf{y}(t)$ is the DOF vector.

The stiffness matrix is assumed to be uncertain and can be expressed by

$$\mathbf{K} = \tilde{\mathbf{K}}(1 + \mathrm{cov}_K \xi_K) \tag{1}$$

other parameters can also be uncertain, for convenience, only stiffness is considered to be uncertain in this section. The parameters in Eq. (1) are shown as follows:ξ_K: standard normal deviate of stiffness, cov_K: cov of stiffness

The system is deterministic when the coefficient of variance (cov) is 0. The simple uncertainty is chosen to show the motivation of this paper. On the basis of the PDD method, the special dynamical characteristics around the deterministic resonant frequency will be explained and highlighted. Therefore, this class of uncertainty will make the system clearer and more convenient to explain. Normal distribution will lead to negative values of the design parameters based on the actually physical significance, so the control parameters are considered to be very small, the corresponding design parameters will be positive. The PDD method can not only be applied in the normal distribution but also in other distributions [25], such as the uniform distribution, beta distribution and so on.

The general dynamical equation can be written as

$$\mathbf{M}\ddot{\mathbf{y}}(t) + \mathbf{C}\dot{\mathbf{y}}(t) + \mathbf{K}\mathbf{y}(t) = \mathbf{F}(t) \tag{2}$$

The external excitation is assumed to be harmonic $\mathbf{F}(t) = \mathbf{F}_0 e^{i\omega t}$, and the steady state response of the system is considered as $\mathbf{y}(t) = \mathbf{Y}e^{i\omega t}$, where $i = \sqrt{-1}$, and $\mathbf{Y}$ is the solution of the following equation:

$$(-\omega^2\mathbf{M} + i\omega\mathbf{C} + \mathbf{K})\mathbf{Y} = \mathbf{F}_0 \tag{3}$$

$\mathbf{K}$ and $\mathbf{Y}$ are the random stiffness matrix and vector respectively, which can be described by the moments of the system. The mean and standard deviation (SD) are calculated, and the calculating formulas of mean and SD are provided in Ref. [29]. The amplitude of the Eq. (3) is $|\mathbf{Y}_1 + i\mathbf{Y}_2|$, $\mathbf{Y}_1$ and $\mathbf{Y}_2$ are the real and imaginary part of $\mathbf{Y}$.