Mechanical Systems and Signal Processing 114 (2019) 106–119

Contents lists available at ScienceDirect

Mechanical Systems and Signal Processing

journal homepage: www.elsevier.com/locate/ymssp

Full-field response monitoring in structural systems driven by a set of identified equivalent forces

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A R T I C L E   I N F O

Article history:
Received 24 October 2017
Received in revised form 26 April 2018
Accepted 2 May 2018

Keywords:
Response prediction
Controllability
Equivalent forces
Kalman filter
Joint input-state estimation
Structural health monitoring

A B S T R A C T

Kalman-type filters for coupled input-state estimation can be used to estimate the full-field dynamic response of structures from only a limited set of vibration measurements. The use of these coupled estimators allows for response prediction to be performed in the absence of any knowledge of both the dynamic evolution and spatial distribution of the excitation forces, where often a set of response-driving equivalent forces is identified from the measurements. In this contribution, a rigorous analysis of the concept of equivalent force based response monitoring is performed, with the aim to clearly establish its limitations and ranges of applicability. It is shown that, unlike commonly assumed, the success of this type of response monitoring cannot be related solely to whether the chosen set of equivalent forces satisfy the controllability requirements, but will depend on the bandwith of the excitation forces in combination with the extent/characteristics of the sensor network. Arguments are instantiated using simple numerical examples where a comparison is made between the theoretical assumptions used to derive the filters and the physical situation. Included in the analyses are situations where (a) the applied and equivalent loads are concentrated and collocated, (b) the applied and equivalent loads are concentrated and non-collocated, (c) modal equivalent loads are used to represent concentrated non-moving forces, and (d) modal equivalent loads are used to represent concentrated moving forces. Results are applicable to any Kalman-type coupled input-state estimator derived using the principles of minimum-variance unbiased estimation.

1. Introduction

The problem of reconstructing the dynamic states of a system from sparse vibration measurements is considered, thereby contributing to ongoing research in the area of full-field response estimation on the basis of output-only data. The full-field response estimates are most commonly used for the purpose of monitoring fatigue damage accumulation in the entire body of large-scale structures [1–3], but can also be used to obtain valuable insights into complex interaction phenomena, e.g. ice-induced vibrations [4,5], when monitored in conjunction with the applied forces.

In recent years, a large number of inverse methodologies for the extrapolation of locally measured structural response to unmeasured locations – through the estimation of full-field quantities – have been proposed. These methodologies can roughly be divided into two categories, namely those that assume that the numerical model used for the extrapolation is free from errors, and those that do account for modelling errors. Belonging to the former category are the modal expansion

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https://doi.org/10.1016/j.ymssp.2018.05.014
0888-3270/

algorithms, where modal coordinates are identified through a pseudo-inversion of the mode shape matrix. These approaches work well for dynamic systems characterized by relatively low-order dynamic behavior, e.g. the supporting structures of offshore wind turbines [6,7]. The second category consists of various Kalman-type filters, where modelling as well as measurement errors are included in the estimation as stochastic processes of which only the variances are known. Various state, coupled input-state, and coupled input-state-parameter estimation algorithms have been developed in this context, ranging from initial formulations for use with linear systems to alternative filters for dealing with reduced-order models, acceleration-only data and recently also nonlinear model descriptions. Limiting the discussion to linear systems, again two groups of filters can be distinguished: those that rely on some assumptions regarding the loading on the structure (e.g. white noise, or characterized by a certain known variance) [1,8,9], and those that are able to observe full-field structural response in the absence of any knowledge of the dynamic evolution of the applied loading [10–12].

In this contribution, we will focus on the second category presented above, namely the Kalman-type filters, and more specifically on cases in which also the spatial distribution of the loading is assumed unknown, and a set of response-driving equivalent forces is identified from the measurements. This is done when the locations of the main excitation sources are uncertain and/or time-varying, which is often the case for large-scale civil or offshore structures. Through a careful analysis of the assumptions forming the basis for the derivation of these algorithms and their possible violation in various real-life applications, the limitations of response estimation based on the concept of equivalent loading is exemplified. Up to now, the success of this type of equivalent force based response estimation has been related only to whether the chosen driving forces satisfy the controllability requirements [13]. It will be shown that controllability is an insufficient criterion for guaranteeing the accuracy of the response estimates, and that results remain sensitive to a correct assumption on the force locations, unless the sensor network allows for the identification of a sufficiently large number of modal forces. Concepts are illustrated using simple numerical examples where a comparison is made between the true and assumed noise statistics and the response prediction accuracy for a number of distinct cases in which the locations of the forces are only known approximately, or not at all, in which case modal equivalent forces are assumed.

The paper is structured as follows: first the state-space notations which will be required for the definition of the necessary error statistics are presented in Section 2, followed by a summary of the Kalman-type filters to which the results will apply and the assumptions forming the basis for their derivation in Section 3. Section 4 presents the concept of equivalent force based response monitoring, and includes the development of new notation for quantifying discrepancies between the theoretical assumptions used to derive the filters and the true physical situation. The latter notations are then used in a set of numerical examples in Section 5, on which basis conclusions are drawn for a selection of commonly encountered engineering problems in Section 6.

2. Formulation of the state-space equations

The formulations presented in this section can be found in many references; we use the notation from [11], repeated here for convenience. The equations of motion for a linear system discretized in space are formulated as:

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{K}\mathbf{u}(t) = \mathbf{f}(t) = \mathbf{S}_{\mathrm{p}}(t)\mathbf{p}(t) \tag{1}$$

where $\mathbf{u}(t) \in \mathbb{R}^{n_{\mathrm{DOF}}}$ is the vector of displacement, $\mathbf{M}, \mathbf{C}$ and $\mathbf{K} \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{DOF}}}$ denote the mass, damping and stiffness matrix, respectively, and $\mathbf{f}(t)$ is the excitation vector. The excitation is factorized into a force influence matrix $\mathbf{S}_{\mathrm{p}} \in \mathbb{R}^{n_{\mathrm{DOF}} \times n_{\mathrm{p}}}$, and the vector $\mathbf{p}(t) \in \mathbb{R}^{n_{\mathrm{p}}}$ representing the n_{p} force time histories. Each column of the matrix $\mathbf{S}_{\mathrm{p}}$ gives the spatial distribution of the load time history in the corresponding element of the vector $\mathbf{p}$. In the case of a point load, the column of $\mathbf{S}_{\mathrm{p}}$ has only a limited number of non-zero entries corresponding to the distribution of the load over the degrees of freedom of the FE mesh. In Eq. (1) $\mathbf{S}_{\mathrm{p}}$ is shown as being time dependent to allow also for the case of moving loads.

By introducing the state vector $\mathbf{x}(t) \in \mathbb{R}^{n_{\mathrm{s}}},\ n_{\mathrm{s}} = 2n_{\mathrm{DOF}}$:

$$\mathbf{x}(t) = \begin{pmatrix} \mathbf{u}(t) \\ \dot{\mathbf{u}}(t) \end{pmatrix}$$

the second-order equations of motion (1) can be written in first-order state space form:

$$\dot{\mathbf{x}}(t) = \mathbf{A}_{\mathrm{c}}\mathbf{x}(t) + \mathbf{B}_{\mathrm{c}}\mathbf{p}(t) \tag{2}$$

where the system matrices $\mathbf{A}_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{s}} \times n_{\mathrm{s}}}$ and $\mathbf{B}_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{s}} \times n_{\mathrm{p}}}$ are defined as:

$$\mathbf{A}_{\mathrm{c}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix},\ \mathbf{B}_{\mathrm{c}} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1}\mathbf{S}_{\mathrm{p}} \end{bmatrix}$$

and the subscript 'c' denotes continuous time. The classical continuous-time state space description of the vibrating system is subsequently found by supplementing the state Eq. (2) with the so-called observation equation:

$$\mathbf{d}(t) = \mathbf{G}_{\mathrm{c}}\mathbf{x}(t) + \mathbf{J}_{\mathrm{c}}\mathbf{p}(t) \tag{3}$$

in which