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Effect of fractionally damped compliance elements on amplitude sensitive dynamic stiffness predictions of a hydraulic bushing

Luke Fredette, Rajendra Singh *

Acoustics and Dynamics Laboratory, Smart Vehicle Concepts Center, Department of Mechanical and Aerospace Engineering, The Ohio State University, Columbus, OH 43210, USA

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A B S T R A C T

Hydraulic bushings exhibit significant amplitude dependent behavior which cannot be captured with the linear time-invariant system theory. Accordingly, Fredette et al. (2016) have proposed a nonlinear model, but the amplitude sensitivity has not been adequately described as it is affected by multiple inherent design features. To further improve the predictive capability of nonlinear models, this article extends the prior work by including two key dissipation effects within the (elastomeric) fluid compliance chambers. First, the conventional fluid compliance element is replaced by an equivalent mechanical spring representing the nonlinear elasticity of the pumping chambers. Fractional calculus based and friction-type damping elements are added in parallel to the nonlinear spring elements of pumping chambers. Second, improved quasi-linear models are proposed at four sinusoidal excitation amplitudes, demonstrating amplitude sensitivity in model parameters. Third, new nonlinear models are proposed and numerically simulated, predicting dynamic stiffness magnitudes and loss angles at multiple excitation amplitudes. The sensitivity of dynamic properties to the fractional and frictional damping parameters is qualitatively evaluated. Finally, both quasi-linear and nonlinear models are experimentally validated and are found to be superior to the ones in the literature.

1. Introduction

In a recent paper, Fredette et al. [1] developed hydraulic bushing models with multiple nonlinear elements including a measurement based compliance model for the elastomeric chambers. While the previous article offered new insight into the physics of these devices, the results of [1] suggest that there is room for further improvement in dynamic stiffness predictions which are sensitive to the excitation amplitude. A major deficiency of prior work [1] has been a lack of damping formulation for the compliance chambers (containing the hydraulic fluid) although the elastomeric material exhibits significant viscoelasticity which could influence the amplitude sensitive dynamics. Several investigators have studied the low-frequency dynamics of hydraulic bushings, but the majority of the literature has employed the linear time-invariant system principles [2–5], despite significant amplitude sensitivity observed in such devices [1,6–8]. A few researchers have proposed amplitude sensitive models, using either a quasi-linear or nonlinear approach. For instance, Svensson and Håkansson [6]

* Corresponding author.
E-mail address: singh.3@osu.edu (R. Singh).

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developed a hydraulic bushing model which included a nonlinear elastic element for the rubber path, but used the linear system principles for the fluid path. Chai et al. [7] proposed a model with a nonlinear fluid resistance term, which introduced significant amplitude sensitivity. However, this study was limited to a laboratory device which replicated only certain behavior of a production device. A recent experimental study by Fredette et al. [8] identified pressure dependent compliance behavior in the fluid system of a production bushing, which led to improved models with both nonlinear fluid resistance and chamber compliance elements [1,8], but provided limited analysis of the damping.

The major goal of this article is to significantly extend the prior formulation [1] by introducing fractional calculus based viscoelasticity and additional nonlinear elements. New or improved models of this study are expected to enhance modeling capabilities and to better understand the underlying physics of hydraulic bushings since they often have complex designs with interacting features. In particular, the effect of frictionally and fractionally damped compliance chambers will be studied and a new technique for analyzing this potentially nonlinear feature will be proposed. The literature on this topic is limited even though the fractional calculus based constitutive laws have been shown to represent the viscoelasticity of many elastomeric materials in a compact, accurate, and physically meaningful way [9–15]. Such viscoelastic behavior should be present in many types of elastomeric isolators and hydraulic bushings. In particular, the approach of the current article is inspired, in part, by the prior work of Sjoberg and Kari [11] who combined nonlinear elasticity, fractional viscoelasticity, and smoothened dry friction damping to mimic the dynamic behavior of a carbon black filled rubber isolator. Instead, the focus of this article will be on the damping effects of elastomeric materials on certain fluid system elements in a typical hydraulic bushing that is commonly employed in vehicle suspension systems [16,17].

2. Problem formulation

The scope of this article is on a class of production grade hydraulic bushings that is schematically described via a baseline lumped parameter system model in Fig. 1; it is equivalent to the formulation of [1]. Steady-state sinusoidal excitation and transmitted force response are used to estimate the dynamic stiffness $\tilde{K}$ in the example case used in [1]. Fig. 1 shows a schematic of the component split into two parallel force transmission paths, where $x(t) = (x_a/2)\sin(2\pi\Omega t)$ is the inner sleeve's displacement excitation, with peak-to-peak amplitude x_a and frequency Ω with units of Hz. The forces transmitted to the outer sleeve through the rubber and fluid paths are denoted F_r and F_f, respectively. Here, p_1 and p_2 represent the dynamic pressures in each pumping chamber, while q_i denotes the volume flow rate in the inertial track. The fluid resistance and inertance of the inertia track are given by R_i and I_i, and C_f is the fluid compliance of the pumping fluid, uniform in both chambers. The effective pumping area of the inner sleeve is given by A_x. Finally, the rubber path stiffness is denoted k_r, with rubber path damping force defined in a function form as $g_r(x, \dot{x})$. Typically, viscous damping would be employed, implying that $g_r(x, \dot{x}) = \eta_r \dot{x}$, where η_r is the viscous damping coefficient.

The governing equations of the fluid system are given by the continuity equations for each pumping chamber,

$$C_f \dot{p}_1(t) = A_x \dot{x}(t) - A_c \dot{y}_1(t) - q_i(t), \tag{1a}$$

$$C_f \dot{p}_2(t) = -A_x \dot{x}(t) - A_c \dot{y}_2(t) + q_i(t), \tag{1b}$$

and the momentum equation for the inertial track,

$$I_i \dot{q}_i(t) = p_1(t) - p_2(t) - R_i q_i(t). \tag{1c}$$

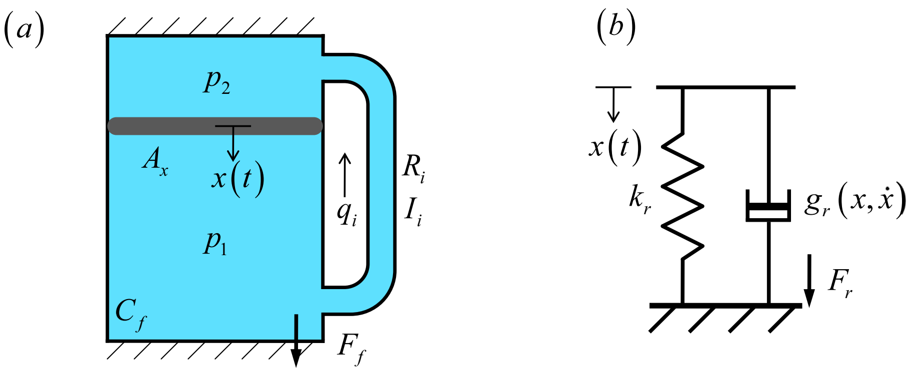

Fig. 1. Schematic of the hydraulic bushing where the (a) fluid path and (b) rubber path subsystems correspond to the analysis of prior work [1]. Here, $x(t)$ is the inner sleeve's displacement excitation while F_r and F_f are the forces transmitted to the outer sleeve through the rubber and fluid paths, respectively. The pressures in each pumping chamber are denoted by p_1 and p_2, and q_i is the volume flow rate in the inertial track. The fluid resistance and inertance of the inertia track are given by R_i and I_i, and C_f is the combined fluid compliance of the pumping fluid and each chamber. A_x is the effective pumping area of the inner sleeve. The rubber path stiffness is denoted k_r, with damping force $g_r(x, \dot{x})$.