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Brief paper

Distributed practical output tracking of high-order stochastic multi-agent systems with inherent nonlinear drift and diffusion terms[☆]

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ABSTRACT

This paper investigates the distributed tracking problem for a class of high-order stochastic nonlinear multi-agent systems where the subsystem of each agent is driven by nonlinear drift and diffusion terms. For the case where the graph topology is directed and the leader is the neighbor of only a small portion of followers, a new distributed integrator backstepping design method is proposed, and distributed tracking control laws are designed, which can effectively deal with the interactions among agents and coupling terms. By using the algebra graph theory and stochastic analysis, it is shown that the closed-loop system has an almost surely unique solution on $[0, \infty)$, all the states of the closed-loop system are bounded in probability, and the tracking errors can be tuned to arbitrarily small with a tunable exponential converge rate. The efficiency of the tracking controller is demonstrated by a simulation example.

1. Introduction

Research on distributed tracking of networked cooperative systems has attracted much attention in the past two decades due to their wide practical applications in areas such as large scale robotic systems (Belta & Kumar, 2002) and biological systems (Olfati-Saber, 2006). The main task of the distributed tracking is to drive the states of the followers to converge to those of a time-varying leader in the circumstance where only a portion of the followers has access to the leader's states and the followers have only local interactions. For this kind of problems, Hu and Hong (2007) and Zhu and Cheng (2010) consider the case with time-varying delays in autonomous agents. Hu and Feng (2010), Huang and Manton (2009) and Ma, Li, and Zhang (2010) consider the case with noises in communication channels. Hong and Wang (2009) and Lou, Hong, and Shi (2012) consider the case with switching topology.

Since all physical systems are nonlinear in nature (Khalil, 2002), it is necessary and beneficial to study the distributed problem in a network of nonlinear dynamical systems. Shi and Hong (2009) consider global target aggregation and state agreement of nonlinear multi-agent systems with switching topologies. Song, Cao, and Yu (2010) present a pinning control and achieves leader-following consensus for multi-agent systems described by nonlinear second-order dynamics. Yu, Chen, and Cao (2011) investigate the consensus issue for the case where the nonlinear intrinsic function is Lipschitz and the directed network is generalized algebraically connected. Meng, Lin, and Ren (2013) study the distributed robust cooperative tracking problem for multiple non-identical second-order nonlinear systems with bounded external disturbances.

Although some progress has been made towards cooperative tracking control of nonlinear multi-agent systems, the existing literature often assumes a simplified system model such as single integrators or double integrators. Also, there are very few results considering stochastic noise. This limits the validity of the models, since stochastic nonlinear systems are ubiquitous in practice. Thus, it is important for us to consider the distributed tracking problem of multi-agent systems with stochastic nonlinear dynamics.

In this paper, the distributed tracking problem of high-order stochastic nonlinear multi-agent systems with inherent nonlinear drift and diffusion terms is investigated under a directed graph

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topology. By using the algebra graph theory and stochastic analysis method, distributed controllers are designed to ensure that the tracking error converges to an arbitrarily small pre-given neighborhood of zero. The main contributions of this paper include:

(1) A new distributed integrator backstepping design is proposed. Different from the traditional integrator backstepping design method used for the single-agent system (Deng & Krstić, 1997; Krstić & Deng, 1998; Li & Wu, 2013; Li, Xie, & Zhang, 2011; Liu, Jiang, & Zhang, 2008; Liu, Zhang, & Jiang, 2007), the distributed tracking control design for nonlinear multi-agent systems needs to consider the interactions among agents, coupling terms in dynamics, and the capability on information collection of each agent and so on, which makes the controller design and performance analysis of the closed-loop systems much more difficult, and new design tools and analysis methods should be introduced.
(2) The systems investigated is high-order, stochastic and with inherent nonlinear drift and diffusion terms. Most of the available results about nonlinear multi-agent systems focus on the dynamics described by single or double integrators (Meng et al., 2013; Shi & Hong, 2009; Song et al., 2010). Recently, Zhang and Frank (2012) investigate the cooperative tracking control of higher-order nonlinear systems with Brunovsky form, in which the first $(M-1)$-dimensional subsystem is linear and the last 1-dimensional subsystem is nonlinear in each agent. However, in this paper, for each agent, all the subsystems are allowed to be nonlinear. Besides, we consider stochastic noises which makes the system model much more general and practical.
(3) The distributed controllers are designed to ensure that the tracking error exponentially converges to an arbitrarily pre-given small neighborhood of zero. The bound of tracking errors and the convergence rate can be explicitly given.

The remainder of this paper is organized as follows. Section 2 is on notation. Section 3 is for problem formulation. Section 4 presents a distributed integrator backstepping design method. Section 5 analyzes the performance properties of the closed-loop systems. Section 6 gives a numerical example to show the effectiveness of the theoretical results. Section 7 includes some concluding remarks.

2. Notation

The following notation will be used throughout the paper. For a given vector or matrix X, X^T denotes its transpose. $\mathrm{Tr}\{X\}$ denotes its trace when X is square, and $|X|$ is the Euclidean norm of a vector X. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},A)$ be a weighted digraph of order n with the set of nodes $\mathcal{V}=\{1,2,\dots,n\}$, set of arcs $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $A=(a_{ij})_{n\times n}$ with nonnegative elements. $(j,i)\in\mathcal{E}$ means that agent j can directly send information to agent i. In this case, j is called the parent of i, and i is called the child of j. The set of neighbors of vertex i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(j,i)\in\mathcal{E},\ i\neq j\}$. $a_{ij}>0$ if node j is a neighbor of node i and $a_{ij}=0$ otherwise. In this paper, we assume that there is no self-loop, i.e. $a_{ii}=0$. Node i is called an isolated node, if it has neither parent nor child. Node i is called a source if it has no parents but children. Denote the sets of all sources and isolated nodes in $\mathcal{V}$ by $\mathcal{V}_s=\{j\in\mathcal{V}|N_j=\emptyset,\ \emptyset \text{ is the empty set}\}$. To avoid the trivial cases, $\mathcal{V}-\mathcal{V}_s\neq\emptyset$ is always assumed in this paper. A sequence $(i_1,i_2),(i_2,i_3),\dots,(i_{k-1},i_k)$ of edges is called a directed path from node i_1 to node i_k. A directed tree is a digraph, where every node except the root has exactly one parent and the root is a source. A spanning tree of $\mathcal{G}$ is a directed tree whose node set is $\mathcal{V}$ and whose edge set is a subset of $\mathcal{E}$. The diagonal matrix $D=\mathrm{diag}(\kappa_1,\kappa_2,\dots,\kappa_n)$ is the degree matrix, whose diagonal elements $\kappa_i=\sum_{j\in\mathcal{N}_i}a_{ij}$. The Laplacian of a weighted digraph $\mathcal{G}$ is defined as $L=D-A$.

We consider a system consisting of n agents and a leader (labeled by 0) which is depicted by a graph $\bar{\mathcal{G}}=(\bar{\mathcal{V}},\bar{\mathcal{E}})$, where $\bar{\mathcal{V}}=\{0,1,2,\dots,n\}$, set of arcs $\bar{\mathcal{E}}\subset\bar{\mathcal{V}}\times\bar{\mathcal{V}}$. If $(0,i)\in\bar{\mathcal{E}}$, then $0\in\mathcal{N}_i$. A diagonal matrix $B=\mathrm{diag}(b_1,b_2,\dots,b_n)$ is the leader adjacency matrix associated with $\bar{\mathcal{G}}$, where $b_i>0$ if node 0 is a neighbor of node i; and $b_i=0$, otherwise.

Definition 1 (*Krstić & Deng, 1998*)**.** A stochastic process $x(t)$ is said to be bounded in probability if $|x(t)|$ is bounded in probability uniformly in t, i.e.,

$$\lim_{c\to\infty}\sup_{t>t_0}P\{|x(t)|>c\}=0.$$

3. Problem formulation

Consider the following high-order stochastic nonlinear multi-agent systems (the followers) with inherent nonlinear drift and diffusion terms described by:

$$\begin{aligned} dx_{ij}&=(x_{i,j+1}+f_{ij}(\bar{x}_{ij}))dt+g_{ij}(\bar{x}_{ij})d\omega, \qquad j=1,\dots,n_i-1,\\ dx_{i,n_i}&=(u_i+f_{i,n_i}(\bar{x}_{i,n_i}))dt+g_{i,n_i}(\bar{x}_{i,n_i})d\omega,\\ y_i&=x_{i1}, \end{aligned}\tag{1}$$

where $\bar{x}_{ij}=(x_{i1},\dots,x_{ij})^T\in R^j$, $u_i\in R$, $y_i\in R$ are the state, input, output of the ith follower, respectively, $i=1,\dots,N$. ω is an m-dimensional independent standard Wiener process defined on the complete probability space $(\Omega,\mathcal{F},\mathcal{F}_t,P)$ with a filtration $\mathcal{F}_t$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all P-null sets). The unknown functions f_{ij} and g_{ij} are smooth with $f_{ij}(0)=0$, $g_{ij}(0)=0$, $i=1,\dots,N$, $j=1,\dots,n_i$.

The following assumptions are made on system (1).

Assumption 1. The unknown functions $f_{ij}(\bar{x}_{ij})$ and $g_{ij}(\bar{x}_{ij})$ are bounded by known nonnegative smooth functions. Specifically, there exist known nonnegative smooth functions $\bar{f}_{ij}(\bar{x}_{ij})$ and $\bar{g}_{ij}(\bar{x}_{ij})$ such that

$$|f_{ij}(\bar{x}_{ij})|\le\bar{f}_{ij}(\bar{x}_{ij}),\qquad |g_{ij}(\bar{x}_{ij})|\le\bar{g}_{ij}(\bar{x}_{ij}).$$

Assumption 2. The leader's output $y_0(t)\in R$ and $\dot{y}_0(t)$ are bounded, and they are only available for the ith follower satisfying $0\in\mathcal{N}_i$, $i=1,\dots,N$.

Assumption 3. The leader is the root of a spanning tree in $\bar{\mathcal{G}}$.

Remark 1. The assumption on the drift term $f_{ij}(\bar{x}_{ij})$ and diffusion term $g_{ij}(\bar{x}_{ij})$ is very general, $i=1,\dots,N$, $j=1,\dots,n_i$. These terms do not need to satisfy global Lipschitz condition (Li, Ren, Liu, & Fu, 2013).

Now, we give the definition for distributed practical output tracking.

Definition 2. The distributed practical output tracking problem for system (1) is solvable if for any given $\varepsilon>0$, there exists a set of distributed control laws such that:

(a) all the states of the closed-loop system are bounded in probability;
(b) for any initial value $x(t_0)$, there is a finite-time $T(x(t_0),\varepsilon)$ such that

$$E|y_i(t)-y_0(t)|^4<\varepsilon,\quad \forall t>T(x(t_0),\varepsilon),\ i=1,\dots,N.$$

The purpose of this paper is to design distributed tracking controllers to solve the distributed practical output tracking problem for system (1).