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Brief paper

Distributed state estimation for linear multi-agent systems with time-varying measurement topology☆

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ABSTRACT

This paper addresses the problem of distributed state estimation in formations of vehicles with time-varying measurement topology. One or more vehicles have access to measurements of their own state, while the rest must rely on measurements relative to other vehicles in the vicinity and limited communication to estimate their own state, and it is assumed that the vehicles may gain or lose access to some measurements over time. The proposed solution consists in the implementation of a local state observer on-board each vehicle, and the effects of changes in the measurement topology on the estimation error dynamics are studied resorting to switched systems theory. Sufficient conditions for exponential stability of the global estimation error dynamics are presented for two different switching laws. The results are applied to the practical case of a formation of Autonomous Underwater Vehicles (AUVs), and simulation results are presented that validate the proposed solution.

1. Introduction

Motivated by the wealth of potential applications for formations composed of multiple agents working cooperatively, see e.g. Bender (1991), Curtin, Bellingham, and Webb (1993), and Giulietti, Pollini, and Innocenti (2000), the subjects of estimation and control in formations of vehicles have been researched extensively in the past few years. The various solutions proposed by the research community can be divided into two very broad categories: centralized and decentralized solutions. Centralized solutions consider the formation as a whole and rely on a central processing node to perform most of the computations, allowing the use of classical, single-vehicle solutions to tackle the multi-agent problem. However, implementation is almost guaranteed to be cumbersome, as heavy computational and communication loads are to be expected due to the necessity of conveying all the information in the formation to a central processing node, which must then relay the results of its computations back to the vehicles. To avoid those pitfalls, the aim of decentralized solutions is to break down the problem in several parts, leaving each agent in the formation with the responsibility of performing a subset of the computations, relying on limited information and communication with other vehicles in the formation. On the subject of decentralized and distributed state estimation, interesting approaches can be found in works such as Barooah (2007), Sousa, Oliveira, and Gaspar (2009), and Yuan and Tanner (2010). The closely related area of distributed control has also seen a wealth of relevant solutions, such as those in Chen, Wen, Liu, and Wang (2014), Fax (2002), and Tanner and Christodoulakis (2007), and recent work on the subject of quadratic invariance has offered optimal solutions for certain classes of measurement topologies, see e.g. Lessard and Lall (2011) and Rokowitz (2008).

The problem addressed in this paper is the design of a distributed state observer for a formation of autonomous vehicles with time-varying measurement topology. One or more vehicles have access to measurements of their own state, while the rest must rely on measurements relative to other vehicles in the vicinity and limited communication with those agents in order to estimate their state. Between switches in the measurement topology, the vehicles only communicate state estimates to use as

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comparison terms for the relative measurements. When there are changes in the measurement topology, the vehicles also need to exchange tables of edges and control vectors with other agents in the vicinity to recover the new measurement graph.

The problem is formulated as a state observer design problem with a sparsity constraint on the output injection gains to reflect the limited amount of information available to each agent. In Viegas, Batista, Oliveira, and Silvestre (2012), the problem was addressed for the fixed topology case, that is, when the measurements available to each vehicle remain the same over time. However, as sensing and communication in formations of vehicles can be unreliable in most practical cases, it is assumed that the measurements available to the agents can change over time, resulting in a time-varying measurement topology. To address this issue, a strategy is outlined for the vehicles to cope with the changes in the measurements, and a switched systems approach is employed to study the stability of the distributed state observer. The error dynamics are formulated as a switched linear system, and sufficient conditions for their exponential stability are derived for two different switching laws. These stability results can be used by mission planners for multi-agent systems as they establish requirements for the behavior of the formation and the sensing and communication equipment of each vehicle that guarantee position and velocity estimates with stable error dynamics over the course of the mission. The behavior of this solution is then assessed in simulation for a formation of Autonomous Underwater Vehicles (AUVs). Preliminary results on this subject can be found in Viegas, Batista, Oliveira, and Silvestre (2013). This paper extends this work with more complete theoretical results, and a more thorough treatment of the transition periods between stable configurations.

The rest of the paper is organized as follows. Section 2 describes the problem at hand and introduces the dynamics of the vehicles and their respective local observers, while Section 3 details the framework necessary to describe the formation-wide dynamics. Section 4 outlines the strategy followed by the vehicles to cope with changes in the measurement topology, and in Section 5 the stability of the error dynamics is analyzed. Section 6 presents simulation results for a formation of AUVs and, finally, Section 7 summarizes the main conclusions of the paper.

1.1. Notation

Throughout the paper the symbol $\mathbf{0}$ denotes a matrix (or vector) of zeros and $\mathbf{I}$ an identity matrix, both of appropriate dimensions. Whenever relevant, the dimensions of an $n \times n$ identity matrix are indicated as $\mathbf{I}_n$. The Kronecker product of two matrices $\mathbf{A}$ and $\mathbf{B}$ is denoted by $\mathbf{A} \otimes \mathbf{B}$. For $x \in \mathbb{R}$, $\lfloor x \rfloor$ represents the largest integer not larger than x.

2. Problem statement

Consider a formation composed of N autonomous vehicles, in which each vehicle is indexed by a distinct positive integer $i \in \{1, 2, \ldots, N\}$, and has sensors mounted on-board which give access to either:

- measurements based on its own state, denoted as "absolute" measurements for convenience; or
- measurements based on its state relative to N_i other vehicles in the vicinity. Furthermore, it is assumed that those vehicles send updated state estimates to vehicle i through communication.

The problem considered in this paper is the design of a distributed state observer that allows each vehicle to estimate its state based primarily on the aforementioned measurements, as well as limited communication between vehicles. The solution proposed here consists in the implementation of a local state observer on-board each vehicle. To achieve a decentralized structure, those local observers must be designed such that, during operation, each vehicle only requires locally available measurements and limited communication with other vehicles in its vicinity to estimate its state. Doing this allows to greatly reduce the communication and computational load in the formation in comparison with a standard centralized implementation, in which all measurements would need to be communicated to a central processing node, which would then perform all the computations and then broadcast the state estimates to the whole formation. This is specially relevant in cases where communication between agents is challenging, such as in underwater applications, or when strict limits on the payload condition the processing, sensing, and communication equipment that can be mounted on-board each vehicle, such as with airborne vehicles.

This section details the dynamics of the vehicles and their respective local state observers, leaving the analysis of the formation-wide dynamics to subsequent sections.

2.1. Local state observer design

For a vehicle i which has access to absolute measurements, its dynamics are described by the Linear Time-Invariant (LTI) system

$$\begin{cases}\dot{\mathbf{x}}_i(t) = \mathbf{A}_L\mathbf{x}_i(t) + \mathbf{B}_L\mathbf{u}_i(t)\\ \mathbf{y}_i(t) = \mathbf{C}_L\mathbf{x}_i(t),\end{cases} \tag{1}$$

where $\mathbf{x}_i(t) \in \mathbb{R}^{n_L}$ is the state of vehicle i, to be estimated, $\mathbf{u}_i(t) \in \mathbb{R}^{m_L}$ is the input of the system, and $\mathbf{y}_i(t) \in \mathbb{R}^{o_L}$ is the output. $\mathbf{A}_L$, $\mathbf{B}_L$, and $\mathbf{C}_L$ are given constant matrices of appropriate dimensions. For this case, it is straightforward to design a Luenberger observer with globally exponentially stable error dynamics, see e.g. Astrom and Murray (2008). On the other hand, if vehicle i has access to relative measurements to N_i other vehicles in the formation, its dynamics follow

$$\begin{cases}\dot{\mathbf{x}}_i(t) = \mathbf{A}_L\mathbf{x}_i(t) + \mathbf{B}_L\mathbf{u}_i(t)\\ \mathbf{y}_i(t) = \mathbf{C}_i\Delta\mathbf{x}_i(t),\end{cases} \tag{2}$$

in which $\mathbf{y}_i(t) \in \mathbb{R}^{o_L \times N_i}$, $\mathbf{C}_i = \mathbf{I}_{N_i} \otimes \mathbf{C}_L$, and

$$\Delta\mathbf{x}_i(t) := \begin{bmatrix}\mathbf{x}_i(t) - \mathbf{x}_{\theta_{i,1}}(t)\\ \mathbf{x}_i(t) - \mathbf{x}_{\theta_{i,2}}(t)\\ \vdots\\ \mathbf{x}_i(t) - \mathbf{x}_{\theta_{i,N_i}}(t)\end{bmatrix} \in \mathbb{R}^{n_L N_i}, \quad \theta_{i,j} \in \Theta_i,$$

where

$$\Theta_i := \left\{\theta_{i,1}, \theta_{i,2}, \ldots, \theta_{i,N_i} | \theta_{i,j} \in \{1, \ldots, N\}\right\}$$

is the set of other vehicles' indexes corresponding to the relative measurements available to vehicle i. For this case, the following local observer structure for the system (2) can be implemented:

$$\begin{cases}\dot{\hat{\mathbf{x}}}_i(t) = \mathbf{A}_L\hat{\mathbf{x}}_i(t) + \mathbf{B}_L\mathbf{u}_i(t) + \mathbf{L}_i(\mathbf{y}_i(t) - \hat{\mathbf{y}}_i(t))\\ \hat{\mathbf{y}}_i(t) = \mathbf{C}_i\Delta\hat{\mathbf{x}}_i(t),\end{cases} \tag{3}$$

where $\mathbf{L}_i \in \mathbb{R}^{n_L \times o_L N_i}$ is a constant matrix of observer gains, to be computed, and $\Delta\hat{\mathbf{x}}_i(t)$ is an estimate of $\Delta\mathbf{x}_i(t)$, computed using the received state estimates:

$$\Delta\hat{\mathbf{x}}_i(t) := \begin{bmatrix}\hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_{\theta_{i,1}}(t)\\ \hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_{\theta_{i,2}}(t)\\ \vdots\\ \hat{\mathbf{x}}_i(t) - \hat{\mathbf{x}}_{\theta_{i,N_i}}(t)\end{bmatrix} \in \mathbb{R}^{n_L N_i}, \quad \theta_{i,j} \in \Theta_i.$$

3. Estimation error dynamics

The structure of the formation can be described using two graphs: a directed measurement graph $\mathcal{G}_M$ and a communication