



Functionally Pooled models for the global identification of stochastic systems under different pseudo-static operating conditions



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ABSTRACT

The problem of identifying a single global model for stochastic dynamical systems operating under different conditions is considered within a novel Functionally Pooled (FP) identification framework. Within it a specific value of a measurable scheduling variable characterizes each operating condition that has pseudo-static effects on the dynamics. The FP framework incorporates parsimonious FP models capable of fully accounting for cross correlations among the operating conditions, functional pooling for the simultaneous treatment of all data records, and statistically optimal estimation. Unlike seemingly related Linear Parameter Varying (LPV) model identification leading to suboptimal accuracy in this context, the postulated FP model estimators are shown to achieve optimal statistical accuracy. An application case study based on a simulated railway vehicle under various mass loading conditions serves to illustrate the high achievable accuracy of FP modelling and the improvements over local models employed within LPV-type identification.

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1. Introduction

Many dynamical systems operate under different conditions that significantly affect their dynamics. Oftentimes, the operating conditions are characterized by one or more measurable variables and remain constant or vary slowly over time, thus having a pseudo-static effect on the dynamics. Typical examples include structural systems vibrating under different loading conditions, such as bridges, sea vessels and trains [1,2], structures vibrating under different environmental (for instance temperature) or boundary conditions [3,4], rotating machinery dynamics under different speeds [5], aircraft dynamics at various altitudes or flight conditions [6,7], and many more.

Abbreviation: AR, X, AutoRegressive, eXogenous; ARX, AutoRegressive with eXogenous excitation; FP-ARX, Functionally Pooled ARX; SPP, Samples per Parameter; LS, Least Squares; ML, Maximum Likelihood; RSS, Residual Sum of Squares; SSS, Signal Sum of Squares; BIC, Bayesian Information Criterion; AIC, Akaike Information Criterion; OLS, Ordinary Least Squares; WLS, Weighted Least Squares

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Nomenclature¹**Important Symbols**

k	scheduling variable	a_{ij}, b_{ij}	AR, X coefficients of projection
$x_k[t]$	excitation signal for the k operating condition	$G_j(k)$	j basis function
$y_k[t]$	response signal for the k operating condition	θ	coefficients of projection vector
$w_k[t]$	model innovations for the k operating condition	$\bar{\theta}$	augmented parameter vector including innovations variance
$e_k[t]$	one-step-ahead prediction error (residuals) for the k operating condition	M	number of excitation-response signal pairs used for FP-ARX identification
\mathcal{N}	normal distribution	N	signal length in samples for each individual operating condition
na, nb	AutoRegressive (AR) and eXogenous (X) orders	\mathcal{B}	backshift operator ($\mathcal{B}^j \cdot u[t] = u[t-j]$)
pa, pb	dimensionality of AR and X functional subspaces (equal to p if $pa=pb$)	\otimes	Kronecker product
$E\{\cdot\}$	statistical expectation	o	subscript designating actual (true) system
$\gamma_w(k, l)$	innovations cross correlation between operating conditions k and l	$\text{plim}_{N \rightarrow \infty}$	probability limit operator
$\sigma_w^2(k)$	innovations variance as a function of the scheduling variable	\xrightarrow{d}	convergence in distribution
		\xrightarrow{p}	convergence in probability
		$\text{Cov}\{\cdot, \cdot\}$	covariance between two random quantities
		$o(x)$	function that tends to zero faster than x

In such cases the problem of identifying a *single global model* of the system, that is a model capable of representing the dynamics under *any* operating condition based on available excitation-response signal pairs, each one corresponding to a *sample operating condition*, is of particular interest and the subject of the present study.

This problem is typically tackled via Linear Parameter Varying (LPV) models [5,8,9]. These are dynamical models with parameters expressed as functions of the measurable variable(s) – referred to as the *scheduling variable(s)* – designating the operating condition. In this context model identification is based on the so-called *local approach* [10–12], the rationale of which is simple and is based on a two-step approach that effectively splits the problem into two distinct subproblems: First a number of *local* (or else frozen) models – each corresponding to a single operating condition for which excitation-response signal pairs are available – are identified using conventional identification techniques [13, ch. 7] (step 1). Second, the identified models are interpolated, typically using orthogonal interpolation functions, in order to provide a single global model [12, pp. 250–251], [14,15] (step 2).

This approach seems reasonable, as a straightforward extension of classical identification. Yet, when viewed within a *stochastic framework* in which the excitation-response signals are stochastic, it leads to suboptimal accuracy. The intuitive explanation for this is simple, and may be readily understood from the fact that the signal pairs are *not* treated as a single entity, but rather in complete isolation of each other in the process of obtaining each local (conventional) model (step 1). This not only neglects potential cross-correlations among the signal pairs, thus resulting into information loss, but additionally leads to an unnecessarily high number of estimated parameters,² thus violating the principle of *statistical parsimony* [13, p. 492] and further leading to increased estimation variance and thus reduced accuracy (lack of *efficiency* in statistical terminology) [13, pp. 560–562]. To these one should also add the errors involved in the subsequent (step 2) interpolation of the obtained local models when constructing the LPV (global) model. The end result is a final, global, LPV model characterized by reduced – that is suboptimal – accuracy.

Recognizing the aforementioned problems that arise within a stochastic context, the present authors and their co-workers have postulated a novel class of *stochastic global models*, referred to as *Functionally Pooled (FP) models*, for the proper global representation of systems and the remedy of the aforementioned weaknesses [16,17]. The class of FP models resembles the form of LPV models, with some of the important differences being that the signal pairs are treated as a single entity, the number of estimated parameters is minimal, potential cross-correlations among the signal pairs are accounted for, and the estimation is accomplished in a single step (instead of two subsequent steps) as necessary for achieving optimal accuracy. The optimal achievable accuracy is analytically established as well.

The FP identification framework is based on three entities (also see Fig. 1):

¹ Important Conventions

Bold-face upper/lower case symbols designate matrix/column-vector quantities, respectively.

Matrix transposition is indicated by the superscript T .

A functional argument in brackets designates function of an integer variable; for instance $x[t]$ is a function of normalized discrete time ($t = 1, 2, \dots$). The conversion from discrete normalized time to analog time is based on $(t-1)T_s$, with T_s standing for the sampling period.

A hat designates estimator/estimate of the indicated quantity; for instance $\hat{\theta}$ is an estimator/estimate of θ .

Tilde designates sample quantity; for instance $\tilde{\sigma}^2$ designates sample variance.

For simplicity of notation, no distinction is made between a random variable and its value(s).

² Equal to the number of local models times the number of model parameters.

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