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Brief paper

Adaptive stabilization of a class of uncertain switched nonlinear systems with backstepping control☆

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ABSTRACT

In this paper, we focus on the problem of adaptive stabilization for a class of uncertain switched nonlinear systems, whose non-switching part consists of feedback linearizable dynamics. The main result is that we propose adaptive controllers such that the considered switched systems with unknown parameters can be stabilized under arbitrary switching signals. First, we design the adaptive state feedback controller based on tuning the estimations of the bounds on switching parameters in the transformed system, instead of estimating the switching parameters directly. Next, by incorporating some augmented design parameters, the adaptive output feedback controller is designed. The proposed approach allows us to construct a common Lyapunov function and thus the closed-loop system can be stabilized without the restriction on dwell-time, which is needed in most of the existing results considering output feedback control. A numerical example and computer simulations are provided to validate the proposed controllers.

1. Introduction

Adaptive stabilization of nonlinear systems has been extensively discussed in the last two decades, *e.g.*, Krstić, Kanellakopoulos, and Kokotović (1995), Lin and Qian (2002) and Marino and Tomei (1995). Recently, many results about analysis and control of switched systems have been developed, see *e.g.*, Branicky (1998), Geromel and Colaneri (2006), Hespanha, Liberzon, and Teel (2008) and Lin and Antsaklis (2009) for stability analysis, and Cheng, Guo, and Wang (2005), Han, Ge, and Lee (2010), Sang and Tao (2012) and Vu and Liberzon (2011) for controller design. Despite these ample results, there are very few in the literature which try to tackle the problem of adaptively stabilizing the switched nonlinear systems with unknown parameters and unknown switching information. The present paper exactly attempts to achieve this goal.

Generally speaking, control of fast time-varying systems is a challenging task in adaptive control field since when parameters change rapidly or abruptly, stability and convergence properties will be affected or destroyed. As stated in Tao (2003): "*For a general linear system with non-small parameter variations including unknown jumping parameters at unknown time instants, new adaptive control designs are yet to be developed.*" Nonetheless, adaptive control of unknown switched systems remains an attractive problem though relatively fewer results are available to date. Some alternative adaptive control schemes for fast time varying systems using multiple models are proposed in Han and Narendra (2012) and Hespanha et al. (2001), where the indirect adaptive control method which estimates the plant parameters using multiple models are discussed. The design philosophy of multiple model adaptive control (MMAC) may be more suitable than the traditional adaptive control for plants with rapid change parameters; however, the stability analysis for unknown switched nonlinear systems is still missing in the existing literature. Instead of estimating the switching parameters and then switching the controller to the corresponding one from the multiple models, in this work, we directly estimate the controller parameters which are some transformed bounds on the switching parameters whereby the system can be stabilized without explicitly considering the changes of plant parameters. Thus, stabilization under arbitrary switching signals can be achieved without the restriction on dwell-time. However, the price is that the plants that can be dealt with are subjected to the constraint that the non-switching part of the switched system should be feedback linearizable.

For the choice of controller structure, state feedback backstepping controllers are designed in Ma and Zhao (2010) and Wu (2009)

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to stabilize the completely known switched nonlinear systems subjected to lower-triangular structure under arbitrary switching signals. For a more general switched nonlinear system with impulses, in Han, Ge, and Lee (2009) the authors proposed a state feedback discontinuous adaptive neural backstepping controller to track the desired output, however, average dwell-time condition is needed to ensure system stability. Among the existing results in the literature, state feedback design is discussed in most of the works and there are relatively fewer results of output feedback control of switched systems. Owing to the lack of information of full states, system parameters, and switching signals, it is challenging to design controllers for the uncertain switched systems using only output information. A new output feedback controller using adaptive high gain observer is designed in Han et al. (2010) to stabilize a general class of nonlinearly parameterized switched systems. However, the persistent dwell-time is needed for stability.

In the present paper, we design adaptive state feedback and adaptive output feedback controllers for a class of uncertain switched nonlinear systems. Design of controllers for uncertain systems using the bound on the parameters or on the norm of the vector functions have been discussed in robust adaptive control such as Lin and Qian (2002) and Marino and Tomei (1993a). Based on the idea of estimating the bound on switching parameters, we design an adaptive state feedback controller, whereas in Ma and Zhao (2010) and Wu (2009), all system parameters are assumed to be known. Then, by incorporating some augmented design parameters, an adaptive output feedback controller is proposed for all relative degree cases. The main contribution of this paper is that we propose the new adaptive controllers such that the considered uncertain switched systems can be stabilized under arbitrary switching signals; whereas in the existing results, dwell-time condition is usually needed. Moreover, as a special case, a general class of switched linear systems can also be stabilized by the proposed controller. It is worth mentioning that there are relatively few results about adaptive output feedback stabilization of unknown switched systems under arbitrary switching signals. By the proposed approach, a common Lyapunov function can be constructed with output feedback.

This paper is organized as follows. Problem formulation is given in Section 2. In Sections 3 and 4, adaptive state feedback and output feedback controllers are proposed respectively for the considered system, and stability analysis under arbitrary switching signals is given. In Section 5, some simulations are provided to validate the design, and the conclusions are given in Section 6.

2. Problem statement

Consider the SISO switched nonlinear system which can be linearly parameterized by the switching parameters $\theta^{\sigma(t)}$ and $\theta_b^{\sigma(t)}$:

$$\begin{aligned}\dot{x} &= F_{\sigma(t)}(x) + \theta_b^{\sigma(t)} g(x)u \\ &= f_0(x) + \sum_{i=1}^{p} \theta_i^{\sigma(t)} f_i(x) + \theta_b^{\sigma(t)} g(x)u \\ y &= h(x), \end{aligned} \tag{1}$$

where $x = [x_1, \ldots, x_n]^T \in \mathcal{R}^n$, $u \in \mathcal{R}$, $\theta^{\sigma(t)} = [\theta_1^{\sigma(t)}, \ldots, \theta_p^{\sigma(t)}]^T \in \mathcal{R}^p$ and $\theta_b^{\sigma(t)} \in \mathcal{R}$ are the unknown piecewise constant switching parameters, and $\sigma(t) \in \mathcal{P} = \{1, 2, \ldots, p_s\}$ is a non-Zeno switching signal which is right continuous. The output function h is smooth with $h(0) = 0$, F_k and g are smooth vector fields with $F_k(0) = 0$, $\forall k \in \mathcal{P}$, and $g(x) \neq 0$ $\forall x \in \mathcal{R}^n$. In this paper, the parameters θ^k and θ_b^k, $k \in \mathcal{P}$, the switching time instants T_k, $k = 1, 2, \ldots$, and the switching index $\sigma(t)$, are all unknown.

The control objective is to design the adaptive state feedback and output feedback control to stabilize the switched system under arbitrary switching signals. The system considered here is assumed to be feedback linearizable in the non-switching part of the dynamics. In the following, $|\cdot|$ denotes the absolute value of a scalar and $\|\cdot\|$ denotes the induced 2-norm of a vector or a matrix.

3. Adaptive state feedback stabilization of switched nonlinear systems

If the considered system (1) satisfies the involutive condition in the non-switching part (that is, f_0 and g are feedback linearizable) and the strict triangularity condition (Marino & Tomei, 1995), then, there exists a function $h_s(x)$ and a *parameter independent* diffeomorphic transformation $z = T_s(x) = (h_s(x), \ldots, L_{f_0}^{n-1} h_s(x))$, $T_s(0) = 0$, such that system (1) can be transformed into the parametric strict-feedback form:

$$\begin{aligned}\dot{z}_j &= z_{j+1} + \sum_{i=1}^{p} \psi_{ji}(Z_j)\theta_i^{\sigma}, \quad 1 \le j \le n-1, \\ \dot{z}_n &= S_1(Z_n) + \theta_b^{\sigma} S_2(Z_n)u + \sum_{i=1}^{p} \psi_{ni}(Z_n)\theta_i^{\sigma}, \end{aligned} \tag{2}$$

where $Z_k := [z_1, \ldots, z_k]^T$, for $1 \le k \le n$, ψ_{ji}, $1 \le j \le n$, $1 \le i \le p$, are smooth functions generated from f_i after coordinate transformation, and $S_1(Z_n) = L_{f_0}^n h_s(x) = L_{f_0}^n h_s(T_s^{-1}(Z_n))$, $S_2(Z_n) = L_g L_{f_0}^{n-1} h_s(x) = L_g L_{f_0}^{n-1} h_s(T_s^{-1}(Z_n))$. Note that $S_1(0) = 0$ and $S_2(Z_n) \neq 0$, $\forall Z_n \in \mathcal{R}^n$, since $\{f_0, g\}$ is feedback linearizable. Details about the transformation, the notations of Lie derivatives, and relevant differential geometric tools can be found in Isidori (1995) and Marino and Tomei (1995).

We will design the adaptive state feedback controller from (2). For an unknown switched system, control performance through adaptively estimating the switching parameters may not be satisfactory since the system is persistently switching. Our design strategy is to find controller parameters, which are some transformed bounds on the switching parameters, to adaptively stabilize the system. A backstepping design for non-switched systems with unknown time-varying parameters is proposed in Marino and Tomei (1993b). Based on the robust adaptive control approach, we design a new state feedback controller for (2). In order to make the problem here more tractable, a common but practical assumption is made for the adaptive state feedback controller:

(A1) θ_b^{σ} for all σ's have the same sign and whose common lower bound is known, that is, $0 < \underline{\theta}_b \le |\theta_b^{\sigma}|$, $\forall t \ge 0$.

Without loss of generality, we assume that θ_b^{σ}, $\sigma \in \mathcal{P}$, are positive. The adaptive state feedback backstepping control is designed by the following cases:

For the case that $n = 1$, $\dot{z}_1 = S_1(z_1) + \theta_b^{\sigma} S_2(z_1)u + \sum_{i=1}^{p} \psi_{1i}(z_1)\theta_i^{\sigma}$. Since ψ_{1i}, $1 \le i \le p$, are smooth functions and $\psi_{1i}(0) = 0$, there exist continuous functions $\bar{\psi}_{1i}(z_1)$ such that $\psi_{1i}(z_1) = \bar{\psi}_{1i}(z_1)z_1$. Moreover, since the parameters lie in a compact set, we can find a smooth function $\alpha_1(z_1)$ and an unknown positive constant l_1 such that

$$\left| \sum_{i=1}^{p} \bar{\psi}_{1i}(z_1)\theta_i^{\sigma} \right| \le l_1 \alpha_1(z_1).$$

Similarly, since $S_1(0) = 0$ and S_1 is smooth, we have $|S_1(z_1)| \le |z_1| W_1(z_1)$ for a smooth function $W_1(z_1) \ge 0$. Let $\hat{l}_1$ denotes the estimate of l_1, and design u such that $u = u_1 = \frac{1}{S_2(Z_1)}(u_{11}^* + u_{12}^*)$, where u_{11}^*, u_{12}^* will be determined based on a Lyapunov design process as shown below. Consider a Lyapunov function candidate $V_1 = \frac{1}{2}(z_1^2 + \tilde{l}_1^2)$, where $\tilde{l}_1 = l_1 - \hat{l}_1$, and evaluate the time derivative