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Diagonal time dependent state space models for modal decomposition of non-stationary signals

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ABSTRACT

This work is devoted to the problem of the decomposition of a non-stationary signal into modal components, for which a methodological approach based on diagonal time-dependent state space models is postulated. In particular, on this paper is shown that the response of a diagonal time-dependent state space models can be cast into a modal form characterized by time-dependent amplitudes and frequencies. Later, a Kalman filter based framework for non-stationary modal decomposition is built on the previously discussed diagonal state space representations. The enhanced performance of the proposed methods is demonstrated on a benchmark test consisting of three non-stationary modal components, and on the modal decomposition and denoising of a *ElectroCardio Graphic* signals from the QT database. The proposed methods constitute a reliable tool for on-line modal decomposition of multi-component non-stationary signals, with results comparable and even better than other state-of-the-art methods.

1. Introduction

Most real-life systems are characterized by time-dependent dynamics, while their resulting dynamic response is non-stationary. The effective analysis of non-stationary signals requires appropriate representation methods capable of accurately describing the evolutionary signal dynamics [1, Ch. 5],[2, Ch. 1],[3]. Amongst the most recognizable representation methods of non-stationary signals are non–parametric Time–Frequency Representations (TFR) and Time–Scale Representations (TSR), where the non-stationary signal is characterized in terms of an infinite set of oscillatory components localized in both time and frequency [1, Ch. 5],[4, Ch. 2]. Modal representations, on the other hand, aim at representing a non-stationary signal by means of the superposition of a finite number of oscillatory components, referred to as *modes*. Unlike the stationary case where the amplitude and frequency of each mode are deemed constant, in the non-stationary case, each mode is associated with time-dependent amplitude and frequency values [5]. In contrast to TFR and TSR where representation is hinged on surfaces on the time-frequency plane, modal representations offer more compact representations of non-stationary processes, while facilitate the extraction of individual signal components.

The problem of calculating the modal representation of a given signal shall hereby be referred to as *modal decomposition*. In a modal representation, the signal may consist of a single modal component (thus called mono-component) or may be constructed from the superposition of several modal components (thus referred to as multi-component). The calculation of the modal decomposition of a mono-component signal reduces to the calculation of the *instantaneous frequency* (IF), *instantaneous phase* (IP) and *instantaneous amplitude* (IA) of the signal [6–8]. In addition, modal decomposition in the case of multi-component signals further requires the isolation of individual modal components and the estimation of their respective instantaneous modal parameters [7,8]. Main difficulties associated with the decomposition of multiple modal components stem from the precise localization and tracking of each evolutionary frequency trajectory in a noisy environment, especially in the case of components with crossing frequencies or in the case of vanishing components.

Among the most widely appraised modal decomposition methods are those based on ridge extraction from TFRs or TSRs. Indeed, for noise-free mono-component signals the IF is located at the maximum of the TFR while the IA is associated with the value at the ridge [9]. Then, estimation of the IF of the modal component translates into the localization of the maximum in a TFR. Nonetheless, in noisy signals, IF estimates are noisy as well and often exhibit discontinuities. To this end, more specialized methods based on curve extraction procedures, such as the Viterbi al-

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gorithm, have been developed [6,8]. In multi-component signals, TFR-based modal decomposition may be achieved by the *peeling* method, where the most dominant modal component is extracted from the TFR and subsequently removed from the signal to extract other modal components [7,8,10]. Otherwise image processing algorithms are also available to extract and separate ridges on TFR images [11]. Nonetheless, these specialized TFR-based modal decomposition methods require heavy processing on time-frequency surfaces, while their performance is bound to the method of calculation of the TFR and a careful parameter selection.

Other modal decomposition methods are based on different types of non–parametric non-stationary representations of the signal. For instance, the *Discrete Wavelet Transform* (DWT) and related filter–bank based methods attempt at separating the signal into frequency bands [12,13]. Otherwise, the *Empirical Mode Decomposition* or *Hilbert–Huang Transform* aims at separating the signal into various non-stationary orthogonal signal components referred to as *Intrinsic Mode Functions* from which IA and IF estimates may be derived through the use of the Hilbert transform [5,13–16]. Other methods achieve a modal decomposition by eigen-analysis of the signal autocovariance matrix. Among these, *Subspace Methods* and the *Karhounen–Loève Transform* (also referred to as Principal Component Analysis, Proper Orthogonal Decomposition or Singular Spectrum Analysis) are available [17–20]. A common problem of these methods is that often the resulting decomposition is characterized by modes that have no relation with physical features of the underlying system. Additionally, these methods cannot resolve effectively the cases of crossing or vanishing modes.

Alternatively, it is possible to define a certain parametric structure associated with a modal form of the signal of interest. A simple example is formed by adaptive notch filters, whose central frequency and bandwidth is adaptively adjusted to the signal properties, while their performance is defined by a forgetting factor, which sets a trade–off between the tracking accuracy and the estimation error [21–23]. More powerful methods attempt at tracking the IF and IA of a single modal component either by adaptive tracking [24,25], or by projecting their values into a functional basis [26]. The latter methods are hinged on non-linear optimization methodologies, and as a result their performance is driven by a correct choice of initial conditions to ensure convergence to a global maximum. In turn, the presence of noise will significantly affect the overall performance of these methods.

Linear time-dependent state space representations and their properties may be used with the purpose of calculating modal decompositions of non-stationary signals. One of such methods is based on a particular type of block-diagonal state space representation, where each second–order block is associated with a modal component with specific IA and IF. In turn, each block is parametrized, through a non-linear relation, by the IF of the modal component. Then, a modal decomposition is achieved by joint estimation of the state vector and the instantaneous frequency by means of the Extended Kalman Filter (EKF) or other non-linear state estimation methods. These methods are commonly referred to as *Kalman Filter Frequency Trackers* (KF-FT) [27–32]. Similarly, an equivalence transformation may be used to map the state space representation associated with a *Time-dependent AutoRegressive* (TAR) model of the signal of interest, into a diagonal state space representation, where each entry of the state vector of the diagonalized system is associated with a modal component, while the entries in the diagonal of the state matrix are associated with the IF of the respective modal components [33,34]. After having selected the number of modal components, state-space based modal decomposition methods can lead to on-line estimates of the IF, IA and the modal components on multi-component signals. In addition, these methods can be modified for the case of vanishing components, by on-line adjustment of the number of modal components.

This work is a further contribution towards the class of modal decomposition methods based on linear time-dependent state-space models. In particular, the main aim of this paper is to study the properties of the response of linear time-dependent state-space systems and to demonstrate how such a response can be associated with a modal representation by the use of equivalence transforms. In that sense, and in contrast to most recent studies aiming at describing the response of a linear time-dependent system in terms of Instantaneous and Harmonic Frequency Response Functions [35,36], the main contribution of this work is to show, through the use of equivalence transforms, that the response of a linear time-dependent state-space system may be characterized by the superposition of narrowband frequency components, which in the case of the natural response take the form of modal components with IA and IF associated with the instantaneous eigenvalues of second-order time-dependent state-space blocks. Furthermore, the analysis undertaken in this work leads to a more general and yet simpler parametrization of second-order linear time-dependent state space blocks, which then can be used in combination with Kalman filters to yield accurate on-line estimates of multiple modal components and their respective IA and IF. Two estimation approaches are considered based either on Joint or Decoupled Kalman filter methods, which lead to the proposed *Joint/Dual Kalman Filter Non-stationary Sinusoid Tracking* (JKF-NST or DKF-NST) methods. The postulated JKF-NST and DKF-NST methods turn out to be a generalization of the well-known KF-FT method, with the main advantage being that the concurrent state and parameter estimation problem involves a milder type of non-linearity compared to that one appearing in KF-FTs.

A Monte Carlo analysis involving a three–component non-stationary signal with mode crossing and fading under noise, and the application on denoising of Electro-Cardio Graphic (ECG) records from the QT database, demonstrate the increased robustness of the proposed methods compared to the KF-FT method and the ridge extraction method described in [8], in terms of tracking accuracy of the modal components and their IA and IF. Moreover, the theoretical results obtained from the analysis of linear time-dependent state space models may be further extended towards the design of improved modal decomposition methods.

The paper is organized as follows: Section 2 provides the main terminology and definitions used throughout the work, Section 3 provides the main contributions of this work, where it is shown how the response of a complex diagonal state space system may be cast into a modal form, Section 4 introduces the second order block diagonal structures to be later used in Section 5 for estimation of modal decompositions via Kalman filter methods. Sections 6 and 7 show a performance analysis of the modal decomposition methods in the decomposition of a three component non-stationary signal featuring mode crossing and fading, and in the denoising of ECG records. Finally, the conclusions of this work are summarized in Section 8.

2. Time-dependent state space representations – main definitions

Consider the (discrete-time) time-dependent state space representation:

$$\boldsymbol{z}[t] = \boldsymbol{A}[t] \cdot \boldsymbol{z}[t-1] + \boldsymbol{b}[t] \cdot u[t] \tag{1a}$$

$$y[t] = \boldsymbol{c}[t] \cdot \boldsymbol{z}[t] + w[t], \qquad w[t] \sim \texttt{NID}\left(0, \sigma_w^2\right) \tag{1b}$$

where $t \in \mathbb{Z}^+$ is the normalized discrete time, $y[t] \in \mathbb{R}$ is the response signal, $\boldsymbol{z}[t] \in \mathbb{R}^M$ is the *state vector*, $u[t] \in \mathbb{R}$ is the *excitation*