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Convergence analysis of a quadratic upper bounded TV regularizer based blind deconvolution

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ABSTRACT

We provide a novel Fourier domain convergence analysis for blind deconvolution using the quadratic upper-bounded total variation (TV) as the regularizer. Though quadratic upper-bounded TV leads to a linear system in each step of the alternate minimization (AM) algorithm used, it is shift-variant, which makes Fourier domain analysis impossible. So we use an approximation which makes the system shift invariant at each iteration. The resultant points of convergence are better – in the sense of reflecting the data – than those obtained using a quadratic regularizer. We analyze the error due to the approximation used to make the system shift invariant. This analysis provides an insight into how TV regularization works and why it is better than the quadratic smoothness regularizer.

1. Introduction

In blind deconvolution a sharp image has to be obtained from a single noisy and blurred observation without the knowledge of the point spread function (PSF). The image observation model is assumed to be a linear shift invariant (LSI) system. The observed image gets corrupted due to noise and blur. Conventional image restoration techniques assume that the blur (PSF) is known, whereas in blind restoration the PSF is taken as an unknown quantity and it is estimated along with the original image.

Blind restoration is an ill-posed problem due to the possibility of multiple solutions and due to the fact that the solution changes by a large amount when there is a small perturbation in the input, due to noise. An ill-posed problem is converted to a well-posed problem using regularizers [1]. A regularizer is an additional term added to the cost function to enforce certain conditions on the acceptable solution, thereby reducing the solution space. Commonly used regularization terms are the H^1-norm (quadratic) that enforces smoothness [2], *TV* norm [3,4] that enables solutions which preserve edges, and wavelet domain regularizer [5–7] which favors solutions with sparse wavelet transforms since natural images are claimed to have a sparse wavelet expansion.

The problem of blind deconvolution is a well studied one, as can be seen from the numerous papers in the image processing literature. Despite this, results on real world images with large image and PSF sizes are rarely satisfactory [8,9]. This is either due to the fact that most of the iterative methods take a long time to converge to an acceptable solution or due to the fact that non-iterative methods do not work for all types of images and PSFs. Most of the solutions given in the literature could be classified as either iterative or non-iterative in nature. Non-iterative methods of deconvolution, proposed in [9,10] are limited in the sense that they could be applied only for images which have a specific spectrum or for PSFs which follow Levi distribution. Iterative methods of blind deconvolution are more common [2,3,11–15] and are

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based on iterative procedures like alternate minimization or expectation maximization. Use of total variation as a regularizer for blind deconvolution is reported in [3,16]. A variational approach to blind deconvolution is given in [17]. Sroubek and Flusser [18] propose a blind deconvolution method in a multichannel framework, which is applicable when multiple observations of a scene is available. In [19] a novel algorithm for blind deconvolution from a pair of differently exposed images is reported. Iterative methods do not assume any properties for the image and use certain general constraints on the PSF, like positivity and symmetry, and hence are applicable to a wider variety of images and PSFs. While using iterative methods it is necessary to check whether these methods converge to a useful solution.

Few papers in the literature address the convergence of iterative methods for blind deconvolution. The work by Chambolle and Lions [20] analyzes the minimization process for the case where the PSF is known. Figueiredo and Bioucas-Dias [21] have studied the sufficiency conditions for convergence in deconvolving Poissonian images, with the PSF assumed to be known. For the blind deconvolution case, the work by Chan and Wong [22] analyzes the convergence of alternate minimization method for the H^1 norm. With a Gaussian approximation to the TV function, [17] shows that the blind deconvolution algorithm converges.

In this paper, we consider a quadratic approximation of *TV* as the regularizer, unlike in [3] where the *TV* without any approximation is used as the image and PSF regularizer. Since the image and the PSF are both unknown, both variables need regularization and *TV* may be used in both the cases [3]. This leads to a cost function having three terms – the quadratic data term and the total variations of the image and the PSF. The data term contains the product of the unknown image and the PSF, and *TV* is a nonlinear function which results in a cost function that is non-linear and non-convex. In order to arrive at an optimum solution for the image and the PSF, we use the alternate minimization (AM) method. In AM a solution is arrived at by keeping one variable fixed and minimizing w.r.t. the other variable, so that at each step the problem reduces to that of a non-blind restoration. In [3], where *TV* is used as regularizer for both the unknowns, variational approach is used to solve the blind deconvolution problem. In a later paper by Chan and Wong [22], a convergence analysis for alternating minimization algorithm for blind deconvolution is reported. But the convergence analysis is restricted to only H^1 norm as it could not be extended to the *TV* norm. In the case of H^1 norm, an analysis is feasible since in each step of the AM iteration, the cost function is quadratic in nature, whereas for *TV* norm this is not true, which makes an analysis difficult.

In order to do the convergence analysis we modify the cost function by replacing the *TV* norm by its quadratic upper bound, following the work of Figueiredo et al. [23]. In [23,24] the upper bounding is used, respectively, for denoising and deconvolution with a known PSF. We use the upper bounding for solving blind deconvolution where the PSF is not known. Using the quadratic upper bound for TV makes the cost function at each step of AM quadratic in nature, though the overall cost function remains non-quadratic. With this modification, we provide a convergence analysis for blind deconvolution using alternate minimization with *TV* as a regularizer. The convergence analysis is done in the transform domain. Though approximating TV by a quadratic function makes the solution process linear at each iteration, the system to be solved is not shift invariant. So we make a further approximation at each iteration to make the system shift invariant. The convergence points reached with these two approximations is better than the convergence points obtained when a quadratic smoothness regularizer is used. We obtain the error incurred due to approximating the system as a shift invariant one. The error analysis gives an insight into the regularization process when TV is used as regularizer.

2. Deconvolution framework

The image formation model used is

$$\underline{y} = H\underline{x} + \underline{n}, \tag{1}$$

where $\underline{y}$ and $\underline{x}$ are $MN \times 1$ vectors obtained by lexicographically (i.e. converting the matrix to a vector by column wise or row wise ordering; we have used row wise ordering) ordering the $M \times N$ observed image y and the original image x, respectively. The noise term $\underline{n}$ is a sample of additive white Gaussian noise and H is the $MN \times MN$ convolution matrix obtained from the point spread function (PSF) h of size $P \times Q$. The PSF matrix is assumed to be of much smaller size than the image. In most practical cases, the PSF is non-negative and exhibits lateral or radial symmetry. These properties of the PSF will be used as constraints along with the constraint that PSF is normalized to unity to prevent any shift in the mean of the image.

The problem of blind deconvolution aims at reconstructing the original image x and the PSF h from the noisy observation y. We propose to estimate $\underline{x}$ and $\underline{h}$ ($PQ \times 1$ vector formed by lexicographically ordering h) using a quadratic data term and a *TV* regularizer for both the image and the PSF. The cost function is given as

$$C(\underline{x}, \underline{h}) = \|\underline{y} - H\underline{x}\|^2 + \lambda_x TV(\underline{x}) + \lambda_h TV(\underline{h}), \tag{2}$$

subject to the constraints

$$h(m,n) = h(-m,-n), \quad h(m,n) \geq 0 \ \forall m,n$$
$$\text{and} \quad \sum_m \sum_n h(m,n) = 1,$$

which come from symmetry, positivity and mean invariance, respectively. λ_x and λ_h are the image and PSF regularization factors, respectively. Total variation of $\underline{x}$, $TV(\underline{x})$ is defined as

$$TV(\underline{x}) = \sum_i \sqrt{(\Delta_i^h x)^2 + (\Delta_i^v x)^2}, \tag{3}$$

where Δ_i^h and Δ_i^v correspond to the horizontal and vertical first order differences at each pixel location, i.e. $\Delta_i^h x = x_i - x_j$ and $\Delta_i^v x = x_i - x_l$ where x_j and x_l are the neighbors to left and above, respectively, of x_i. $TV(\underline{h})$ is defined similarly. It should be noted that (2) is highly nonlinear and this makes the convergence analysis difficult. Figueiredo et al. [23,24] have