Signal Processing 106 (2015) 319–330

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

A method to initialize free parameters in lattice structure of arbitrary-length linear phase perfect reconstruction filter bank☆

Bodong Li [a,b], Xieping Gao [a,b,*]

[a] MOE Key Laboratory of Intelligent Computing & Information Processing, Xiangtan University, Xiangtan 411105, China
[b] College of Information Engineering, Xiangtan University, Xiangtan 411105, China

ARTICLE INFO

Article history:
Received 24 April 2014
Received in revised form
21 July 2014
Accepted 11 August 2014
Available online 19 August 2014

Keywords:
Filter bank
Lattice structure
Linear phase
Perfect reconstruction
Free parameter
Initialization

ABSTRACT

The optimization of the free parameters in the lattice structure of linear phase perfect reconstruction filter bank (LPPRFB) is commonly highly nonlinear, therefore the free parameters should be carefully initialized. Such methods have been published for constrained-length LPPRFB (CLLPPRFB), i.e. LPPRFB with filter length MK, where M is the decimation factor and K is an integer. In contrast to CLLPPRFB, arbitrary-length LPPRFB (ALLPPRFB) [i.e. LPPRFB with filter length $MK+\beta$, where β is an integer between 0 and M] provides more choices of filter banks, offering better trade-off between filter length and filter performance. Nevertheless, as far as we know there is no systematic study on initializing the free parameters in the lattice structure of ALLPPRFB, and the required method cannot be easily generalized from those for CLLPPRFB. To address this issue, an initialization method will be described in this paper that initializes a longer ALLPPRFB by a shorter one, and the length difference between the longer and the shorter filter banks can be any number allowed by an ALLPPRFB system. The efficiency of the proposed method will be shown by design examples and image compression experiments.

1. Introduction

An important type of filter bank is linear phase perfect reconstruction filter bank (LPPRFB). One can efficiently construct LPPRFB by lattice structure [1–16], which yields filter bank with free parameters. Optimization of the free parameters will produce more practical filter bank. Since the optimization is generally highly nonlinear, the free parameters should be carefully initialized.

The initialization can be carried out by setting the initial filter bank (the filter bank associated with the initial values of the free parameters) to be a practical filter bank. For the design examples in [7], Gan et al. set the initial filter bank to be Walsh–Hadamard Transform (WHT) filter bank. A formal initialization was proposed by Liang et al. [9], which set the initial filter bank to be DCT filter bank. Muramatsu et al. [14,16] described an initialization, which set the initial filter bank with filter length MK to be a practical filter bank with filter length $M(K-2)$, where M is the decimation factor and K is an integer. The three results all followed the way that initialized a higher-order filter bank by a lower-order one; and the lower-order filter bank was restricted with order of zero [7,9], or the order difference (called increasing order) between the higher

☆ This work was supported by the NSFC (Nos. 61172171, 61302182).
* Corresponding author.
E-mail addresses: bodong.li@gmail.com (B. Li), xpgao@xtu.edu.cn (X. Gao).

http://dx.doi.org/10.1016/j.sigpro.2014.08.016
0165-1684/

order and the lower order was limited to be even integer [14,16]. In 2014, we [17] presented an initialization also following the way that initializes a higher-order filter bank by a lower-order one, yet the order of the lower-order filter bank can be any integer allowed by an LPPRFB system, instead of zero only. Besides, the increasing order can be any allowed integer in an LPPRFB system, rather than even integer only.

Notice that the above-mentioned initializations are all used for constrained-length LPPRFB (CLLPPRFB), i.e. LPPRFB with filter length MK, where M is the decimation factor as mentioned before and K is an integer. Another type of LPPRFB is arbitrary-length LPPRFB (ALLPPRFB), i.e. LPPRFB with filter length $MK+\beta$, where β is an integer between 0 and M. Compared with CLLPPRFB, ALLPPRFB supplies more choices of filter banks and provides better trade-off between filter length and filter performance. However, systematic methods have not been published to initialize the free parameters in the lattice structure of ALLPPRFB, and cannot be easily generalized from those for CLLPPRFB.

In this paper, a method will be proposed to initialize the free parameters in the lattice structure of ALLPPRFB. The method is carried out by initializing a longer ALLPPRFB to be a shorter one, and the length difference (called increasing length) between the longer and the shorter filter banks can be any allowed number in an ALLPPRFB system. Design examples and image compression experiments show that the filter banks produced by the proposed method are comparative or even better compared with published ones.

Notations: The symbols $\mathbf{I}$, $\mathbf{J}$, and $\mathbf{0}$ are reserved for identity matrix, exchange matrix, and null matrix respectively, and the subscripts will be given if necessary. The symbol $m=\lfloor M/2\rfloor$ and $b=\lfloor \beta/2\rfloor$, where $\lfloor x\rfloor$ denotes the floor of the real number x. Besides,

$$\mathbf{W}_{2m}=\begin{bmatrix}\mathbf{I}_m & \mathbf{I}_m\\ \mathbf{I}_m & -\mathbf{I}_m\end{bmatrix},\quad \mathbf{W}_{2m+1}=\begin{bmatrix}\mathbf{I}_m & & \mathbf{I}_m\\ & \sqrt{2} & \\ \mathbf{I}_m & & -\mathbf{I}_m\end{bmatrix}.$$

2. Preliminaries

To construct an M-channel filter bank with each filter of length $MK+\beta$, lattice structure represents its polyphase matrices as the product of building blocks. The polyphase matrices include analysis polyphase matrix $\mathbf{E}(z)$ and synthesis polyphase matrix $\mathbf{R}(z)$, and the latter can be denoted by the former therefore only the former (i.e. $\mathbf{E}(z)$) is considered in this paper. The analysis polyphase matrix is defined as $\mathbf{E}(z)=(E_{i,l}(z);i,\ l=0,\dots,M-1)$ with $E_{i,l}(z)=\sum_{k=0}^{K_0-1}h_i(Mk+l)z^{-k}$, where $h_i(n)$ is the filter coefficient of the analysis filter $H_i(z)=\sum_{n=0}^{KM+\beta-1}h_i(n)z^{-n}$. Here $K_0=K$ for $l<\beta$ and $K_0=K+1$ for $l\geq\beta$, and $K-1$ is the order of the system. The system becomes an ALLPPRFB if the polyphase matrices provide linear phase and perfect reconstruction properties, and can be designed as follows [5,15].

If M is even, then β must be even [15] and we have

$$\mathbf{E}(z)=\mathbf{G}_{K-1}(z)\mathbf{G}_{K-2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0(z) \tag{1}$$

where

$$\mathbf{G}_k(z)=\tfrac{1}{2}\ \mathrm{diag}(\mathbf{I}_m,\mathbf{V}_k)\mathbf{W}_M\ \mathrm{diag}\left(\mathbf{I}_m,z^{-1}\mathbf{I}_m\right)\mathbf{W}_M \tag{2}$$

$$\begin{aligned}\mathbf{E}_0(z)=&\frac{\sqrt{2}}{2}\,\mathrm{diag}(\mathbf{U}_0,\mathbf{V}_0)\mathbf{W}_M\mathrm{diag}(\mathbf{I}_m,\mathbf{J}_m)\\ &\cdot\mathrm{diag}\left(\mathbf{I}_b,\mathbf{I}_{M-\beta},z^{-1}\mathbf{I}_b\right)\\ &\cdot\begin{bmatrix}\mathbf{I}_b & & \\ & & \mathbf{I}_{M-\beta}\\ & \mathbf{I}_b & \end{bmatrix}\mathrm{diag}\left(\mathbf{I}_b,\mathbf{J}_b,\mathbf{I}_{M-\beta}\right)\\ &\cdot\mathrm{diag}\left(\mathbf{W}_\beta,\mathbf{I}_{M-\beta}\right)\ \mathrm{diag}\left(\frac{1}{2}\mathbf{I}_b,\frac{1}{2}\mathbf{N}_0,\mathbf{I}_{M-\beta}\right)\\ &\cdot\mathrm{diag}(\mathbf{W}_\beta,\mathbf{I}_{M-\beta})\ \mathrm{diag}(\mathbf{I}_b,\mathbf{J}_b,\mathbf{I}_{M-\beta}).\end{aligned} \tag{3}$$

The free invertible matrices $\mathbf{V}_i$, $\mathbf{U}_0$, and $\mathbf{N}_0$ have sizes m, m, and b respectively.

If M is odd and β is even, then K must be odd [15] and we get

$$\mathbf{E}(z)=\mathbf{G}_{(K-1)/2}(z)\mathbf{G}_{(K-3)/2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0(z) \tag{4}$$

where

$$\begin{aligned}\mathbf{G}_k(z)=&\tfrac{1}{4}\ \mathrm{diag}(\mathbf{U}_k,\mathbf{I}_m)\mathbf{W}_M\ \mathrm{diag}\left(\mathbf{I}_{m+1},z^{-1}\mathbf{I}_m\right)\mathbf{W}_M\\ &\cdot\mathrm{diag}(\mathbf{I}_{m+1},\mathbf{V}_k)\mathbf{W}_M\ \mathrm{diag}(\mathbf{I}_m,z^{-1}\mathbf{I}_{m+1})\mathbf{W}_M\end{aligned} \tag{5}$$

$$\begin{aligned}\mathbf{E}_0(z)=&\frac{\sqrt{2}}{2}\,\mathrm{diag}(\mathbf{U}_0,\mathbf{V}_0)\mathbf{W}_M\ \mathrm{diag}\left(\mathbf{I}_{m+1},\mathbf{J}_m\right)\\ &\cdot\mathrm{diag}\left(\mathbf{I}_b,\mathbf{I}_{M-\beta},z^{-1}\mathbf{I}_b\right)\\ &\cdot\begin{bmatrix}\mathbf{I}_b & & \\ & & \mathbf{I}_{M-\beta}\\ & \mathbf{I}_b & \end{bmatrix}\mathrm{diag}\left(\mathbf{I}_b,\mathbf{J}_b,\mathbf{I}_{M-\beta}\right)\\ &\cdot\mathrm{diag}\left(\mathbf{W}_\beta,\mathbf{I}_{M-\beta}\right)\mathrm{diag}\left(\frac{1}{2}\mathbf{I}_b,\frac{1}{2}\mathbf{N}_0,\mathbf{I}_{M-\beta}\right)\\ &\cdot\mathrm{diag}(\mathbf{W}_\beta,\mathbf{I}_{M-\beta})\ \mathrm{diag}(\mathbf{I}_b,\mathbf{J}_b,\mathbf{I}_{M-\beta}).\end{aligned} \tag{6}$$

The free invertible matrices $\mathbf{U}_i$, $\mathbf{V}_i$, and $\mathbf{N}_0$ are in size $m+1$, m, and b, respectively.

If M is odd and β is odd, then K must be even [15] and one can obtain

$$\mathbf{E}(z)=\mathbf{G}_{(K-2)/2}(z)\mathbf{G}_{(K-4)/2}(z)\cdots\mathbf{G}_1(z)\mathbf{E}_0(z) \tag{7}$$

where

$$\begin{aligned}\mathbf{G}_k(z)=&\tfrac{1}{4}\mathrm{diag}(\mathbf{U}_k,\mathbf{I}_m)\mathbf{W}_M\ \mathrm{diag}\left(\mathbf{I}_{m+1},z^{-1}\mathbf{I}_m\right)\mathbf{W}_M\\ &\cdot\mathrm{diag}(\mathbf{I}_{m+1},\mathbf{V}_k)\mathbf{W}_M\ \mathrm{diag}(\mathbf{I}_m,z^{-1}\mathbf{I}_{m+1})\mathbf{W}_M\end{aligned} \tag{8}$$

$$\begin{aligned}\mathbf{E}_0(z)=&\frac{\sqrt{2}}{2}\mathrm{diag}(\mathbf{U}_0,\mathbf{V}_0)\mathbf{W}_M\ \mathrm{diag}\left(\mathbf{I}_{m+1},\mathbf{J}_m\right)\\ &\cdot\mathrm{diag}\left(\mathbf{I}_{m+1},z^{-1}\mathbf{I}_m\right)\\ &\cdot\begin{bmatrix}\mathbf{I}_m & & \\ & & 1\\ & \mathbf{I}_m & \end{bmatrix}\mathrm{diag}\left(\mathbf{I}_m,\mathbf{J}_m,1\right)\mathrm{diag}(\mathbf{W}_{2m},1)\\ &\cdot\mathrm{diag}\left(\frac{1}{2}\mathbf{I}_m,\frac{1}{2}\mathbf{N}_0,1\right)\mathrm{diag}(\mathbf{W}_{2m},1)\mathrm{diag}\left(\mathbf{I}_m,\mathbf{J}_m,1\right)\\ &\cdot\mathrm{diag}\left(z^{-1}\mathbf{I}_b,\mathbf{I}_{M-b-1},z^{-1}\right)\\ &\cdot\begin{bmatrix}\mathbf{I}_b & & & \\ & & & \mathbf{I}_{M-\beta}\\ & & \mathbf{I}_b & \\ & 1 & & \end{bmatrix}\mathrm{diag}\left(\mathbf{I}_{b+1},\mathbf{J}_b,\mathbf{I}_{M-\beta}\right)\end{aligned}$$