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Fast communication

Interior point method for optimum zero-forcing beamforming with per-antenna power constraints and optimal step size

Hai Huyen Dam [a,*], Antonio Cantoni [b]

[a] Department of Mathematics and Statistics, Curtin University Perth, Australia
[b] School of Electrical, Electronic and Computer Engineering, The University of Western Australia, Perth, Australia

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ABSTRACT

This paper proposes a new computational procedure for solving the optimal zero-forcing beamforming problem in multiple antenna channels that maximizes user achievable rate with restriction on the per-antenna element power constraints. An interior point method with optimal step size procedure is developed in which the step size for the line search in the Newton search direction is calculated exactly for each iteration. This significantly enhances the efficiency associated with the line search. Design examples show that the proposed algorithm converges rapidly to the optimal solution with low computational complexity.

1. Introduction

The challenges of providing broadband services in the rural area can be tackled through the utilization of a range of communication techniques, which include advanced adaptive multicarrier modulation and coding; QOS based on cross-layer scheduling; and multi-user multiple-input multiple-output (MU-MIMO) systems to increase spectral efficiency. There has been much research on MU-MIMO in multipath environments that naturally provide the channel diversity exploited by MU-MIMO [1–8].

In this paper we consider a MU-MIMO that uses a reduced complexity linear precoding technique called zero forcing beamformer (ZFBF) to serve multiple users [4–8]. Each user stream is coded independently and multiplied by a beamforming weight vector for transmission through multiple BS antennas. ZFBF has been shown to achieve a large fraction of Dirty Paper Coding (DPC) capacity when the base station has multiple antennas and each user has a single antenna. We investigate the optimization of ZFBF in multiple antenna broadcast channel that maximizes user achievable rate with restriction on the per-antenna element power constraints for a rural scenario. In a practical system, optimal weight vectors would have to be generated quickly and hence a low complexity rapidly converging algorithm is required.

In [9–11], the primal–dual interior point method was developed for the zero-forcing beamforming problem with per-element power constraints. The backtracking line search algorithm is used to calculate the step size for each iteration of Newton's search method [12]. As the backtracking algorithm requires multiple calculation of the objective function [11], we develop an algorithm to estimate an exact optimal step size in each iteration. This significantly enhances the efficiency associated with the line search. In addition, we exploit further the special structure of the zero-forcing beamforming problem to reduce the complexity of calculating the updates in each

* Corresponding author. Tel.: +61 8 9266 7670; fax: +61 8 9266 3197.
E-mail addresses: H.dam@curtin.edu.au (H.H. Dam), antonio.cantoni@uwa.edu.au (A. Cantoni).

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iteration of Newton's method. As such, the key contribution of the paper is a new fast computational procedure employing an exact optimal step size calculation for each iteration for the barrier interior point method using Newton's method. Design examples show that the proposed algorithm converges fast to the optimal solution with low computational complexity.

The outline of the paper is given as follows. Section 2 introduces the system model while Section 3 formulates the optimization problem. Section 4 discusses the simulation results while Section 5 concludes the paper.

2. System model

Consider a system equipped with M transmit antennas and N receiver antennas. For $1 \le n \le N$, the received signal y_n for the nth user antenna is

$$y_n = \mathbf{h}_n^H \mathbf{s} + v_n$$

where $\mathbf{h}_n$ is the nth user antenna complex channel vector, $\mathbf{h}_n = [h_{n1}, \dots, h_{nM}]^T$, $\mathbf{s}$ denotes the antenna outputs, $\mathbf{s} = [s_1, \dots, s_M]^T$, and v_n is the complex Gaussian noise with mean 0 and variance σ^2. Assume that the transmitter employs linear beamforming where the weight vector $\mathbf{w}_n = [w_{n1}, \dots, w_{nM}]^T$ is used to map the nth user data symbol b_n to the antenna outputs. The antenna output $\mathbf{s}$ composed of signals for all N antennas is given by $\mathbf{s} = \sum_{n=1}^N b_n \mathbf{w}_n$.

In the case of zero-forcing beamforming, user antennas do not suffer interference from each other's transmissions because the beamforming vector $\mathbf{w}_k$ is chosen to be orthogonal to all other user channels: $\mathbf{h}_k^H \mathbf{w}_n = 0$, $k \ne n$, $1 \le k, n \le N$. Assume that the data symbols b_n are independent user-to-user and have an i.i.d. Gaussian distribution with zero mean and unit variance. Then, the achievable rate for user antenna n is given by

$$r_n = \log_2\left(1 + \frac{|\mathbf{h}_n^H \mathbf{w}_n|^2}{\sigma^2}\right). \tag{1}$$

For any set of weight vectors the transmit power is required to be at most a specified value P_{max} given by $E[|x_m|^2] \le P_{max}, 1 \le m \le M$. Since the data symbols for different users are independent and identically distributed, the above constraints can also be written as

$$\sum_{n=1}^N |w_{nm}|^2 \le P_{max}, \quad 1 \le m \le M. \tag{2}$$

3. Optimization problems

It is assumed that the channel vectors can be estimated perfectly. Typically this is achieved by using training sequences in the signal that is transmitted which can be used to identify the channels. The problem determines the weight vectors $\{\mathbf{w}_1, \dots, \mathbf{w}_N\}$ that maximize the minimum user achievable rate subject to zero-forcing condition and per-element power constraints. This problem is not a convex optimization problem. However, it can be transformed into a convex problem by observing that the optimum beamforming vectors are invariant to phase-shifts, i.e. if $\mathbf{w}_n^*$ is an optimum beamforming vector, then the vector $e^{j\theta}\mathbf{w}_n^*$ is also an optimum beamforming vector [8]. Thus, if an optimum solution exists, then it is possible to rotate this solution to obtain an equivalent solution so that $\mathbf{h}_n^H \mathbf{w}_n$ is a real number for all n. This results in the following equivalent optimization problem:

$$\begin{cases} \max\limits_{\mathbf{w}_n} & t \\ \text{s.t.} & \mathbf{h}_n^H \mathbf{w}_n \ge t, \quad 1 \le n \le N \\ & \mathcal{I}\{\mathbf{h}_n^H \mathbf{w}_n\} = 0, \quad 1 \le n \le N \\ & \mathbf{h}_k^H \mathbf{w}_n = 0, \quad k \ne n, \ 1 \le k, \ n \le N \\ & \sum\limits_{n=1}^N |w_{nm}|^2 \le P_{max}, \quad 1 \le m \le M. \end{cases} \tag{3}$$

where $t = \sqrt{(2^r - 1)\sigma^2}$. Here, $\mathcal{I}\{\cdot\}$ denotes the imaginary part of and $\mathcal{R}\{\cdot\}$ denotes the real part of a complex number.

Now, we consider the linear constraints in (3). Denote by $\mathbf{w}$ the $2NM \times 1$ vector of real coefficients, $\mathbf{w} = [\mathcal{R}\{\mathbf{w}_1^T\}\mathcal{I}\{\mathbf{w}_1^T\} \dots \mathcal{R}\{\mathbf{w}_N^T\}\mathcal{I}\{\mathbf{w}_N^T\}]^T$. The linear constraint in (3) can be expressed as $\mathbf{A}\mathbf{w} = \mathbf{0}$ where $\mathbf{A}$ is $(2N^2 - N) \times 2NM$ matrix. Similarly, the inequality linear constraints in (3) can be rewritten as $\mathbf{b}_n^T \mathbf{w} + t \le 0, 1 \le n \le N$, where $\mathbf{b}_n = [\mathbf{0} \cdots -\mathcal{R}\{\mathbf{h}_n^T\} - \mathcal{I}\{\mathbf{h}_n^T\} \cdots \mathbf{0}]^T$. Also, the quadratic constraints in (3) can be written in the matrix form as $\mathbf{w}^T \mathbf{C}_m \mathbf{w} \le P_{max}$, $1 \le m \le M$, where $\mathbf{C}_m$ is a diagonal matrix. The optimization problem (3) can be rewritten as

$$\begin{cases} \min\limits_{\mathbf{w}} & -t \\ \text{s.t.} & \mathbf{A}\mathbf{w} = \mathbf{0} \\ & \mathbf{b}_n^T \mathbf{w} + t \le 0, \quad 1 \le n \le N \\ & \mathbf{w}^T \mathbf{C}_m \mathbf{w} \le P_{max}, \quad 1 \le m \le M. \end{cases} \tag{4}$$

The problem (4) is solved by using the barrier interior point method using Newton's method. To reduce the computational complexity associated with the algorithm, the linear constraints are removed to reduce the dimension of the problem. Denote by D the nullity of the matrix $\mathbf{A}$, $D < 2MN$, and $\mathbf{P}$ the $2NM \times D$ matrix whose columns form an orthonormal basis of the null space of the matrix $\mathbf{A}$, $\mathbf{P}^T\mathbf{P} = \mathbf{I}$. The matrix $\mathbf{P}$ can be obtained from the matrix $\mathbf{A}$ by employing a singular value decomposition (SVD) or QR factorization [13]. Hence, any vector $\mathbf{w}$ in the null space of the matrix $\mathbf{A}$ can be mapped to a vector $\mathbf{r}$ in the reduced space with dimension $D \times 1$ and vice versa via the linear transformation

$$\mathbf{r} = \mathbf{P}^T \mathbf{w} \quad \text{and} \quad \mathbf{w} = \mathbf{P}\mathbf{r}. \tag{5}$$

By using the transformation matrix $\mathbf{P}$, the problem (4) can be reformulated as

$$\begin{cases} \min\limits_{\mathbf{r}} & -t \\ \text{s.t.} & \mathbf{b}_{n1}^T \mathbf{r} + t \le 0, \quad 1 \le n \le N \\ & \mathbf{r}^T \mathbf{P}^T \mathbf{C}_m \mathbf{P}\mathbf{r} \le P_{max}, \quad 1 \le m \le M \end{cases} \tag{6}$$

where $\mathbf{w} = \mathbf{P}\mathbf{r}$ and $\mathbf{b}_{n1} = \mathbf{P}^T \mathbf{b}_n$. Let $\mathbf{x} = [\mathbf{r}; t]$ and $\tilde{\mathbf{b}}_n = [\mathbf{b}_{n1}^T 1]^T$. The inequality constraints in (6) can be rewritten in the form $f_m(\mathbf{x}) \le 0$, $1 \le m \le N + M$, where

$$\begin{aligned} f_n(\mathbf{x}) &= \tilde{\mathbf{b}}_n^T \mathbf{x}, \quad 1 \le n \le N \\ f_{m+N}(\mathbf{x}) &= \mathbf{r}^T \mathbf{P}^T \mathbf{C}_m \mathbf{P}\mathbf{r} - P_{max}, \quad 1 \le m \le M. \end{aligned} \tag{7}$$