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Brief paper

# Equal distribution of satellite constellations on circular target orbits☆

Ewoud Vos [1], Jacquelien M.A. Scherpen, Arjan J. van der Schaft

*University of Groningen, Faculty of Mathematics and Natural Sciences, Nijenborgh 4, 9747AG Groningen, The Netherlands*

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ABSTRACT

This paper addresses the problem of equal distribution of satellite constellations on circular target orbits. The control goal is to make the constellation converge to a circular target orbit, while spatially distributing the satellites at equal inter-satellite distances. The solution is defined in the port-Hamiltonian framework, which gives a clear physical interpretation of the obtained control laws, insight into the energy consumption and complete stability proofs. The controller consists of two parts: the internal control system steers each individual satellite to the target orbit, the external control system equally distributes the satellite constellation. Numerical simulation results are given to illustrate the effectiveness of the approach.


## 1. Introduction

Formation flying of satellite constellations has received quite some attention in recent years. There have been different definitions of the terms satellite constellation and formation flying, for example based on the fact whether the states of the satellites are coupled (formation flying Scharf, Ploen, & Hadaegh, 2003) or not (constellation Scharf, Hadaegh, & Ploen, 2004). In this work a satellite constellation refers to a group of satellites which collaborate in order to achieve a higher level goal. By formation flying we refer to this higher level goal itself, which in this work corresponds to achieving certain desired relative distances between the satellites in the constellation.

Using satellite constellations opens up possibilities for new types of missions, which are not possible with the traditional one satellite setup (Das & Cobb, 1998). For example, the OLFAR mission aims at exploring the below 30 MHz frequency bandwidth radio signals. To achieve sufficient spatial resolution, such a low frequency telescope in space needs an aperture diameter of 10–100 km.[2] Clearly, these type of applications are not feasible with a single satellite.

The dynamic environment where constellation operates can be divided into two regimes (Scharf et al., 2003), namely deep space and planetary orbits. In deep space the relative dynamics of a constellation are usually approximated, often by a double integrator. For planetary orbits on the other hand the constellation dynamics are considered explicitly, including gravitational forces and disturbances such as drag. In this work we focus on one particular type of planetary orbit, namely circular orbits.

Much formation flying research has focused on the case where only one point in the formation (e.g. the center of mass or the leader satellite) is on the planetary orbit (Chung, Ahsun, & Slotine, 2009; Kristiansen & Nicklasson, 2009), while the individual satellites do not need to be on the orbit. In contrast, McInnes (1995), Ulybyshev (1998) and this work address the problem where each satellite is on the orbit, while the satellites equally distribute spatially on the orbit. This problem is also known as *orbital phasing*, since the control goal is to keep spacecrafts phased on the orbit (Scharf et al., 2004). Equal spatial distribution on circular orbits is of special interest to Global Navigation Satellite Systems (GNSS) like the Global Positioning System (GPS) and more recently Galileo. GPS requires 24 satellites to equally distribute on six circular orbits, while Galileo requires 30 satellites to distribute on three orbits. Other

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*E-mail addresses:* e.vos@rug.nl (E. Vos), j.m.a.scherpen@rug.nl (J.M.A. Scherpen), a.j.van.der.schaft@rug.nl (A.J. van der Schaft).

1 Tel.: +31 50 363 4771; fax: +31 50 363 3800.

2 http://ens.ewi.tudelft.nl/Research/array/olfar/intro.php.

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applications may be found in meteorological, environmental and military applications.

In this work we propose a controller based on energy considerations to solve the spatial distribution problem. Recently we have witnessed an increasing interest in these so-called energy-based models (Ortega, van der Schaft, Mareels, & Maschke, 2002), which allow for analysis and control design for nonlinear, multi-domain systems such as satellite constellations. The energy function of a system determines not only the static, but also the transient behavior (Ortega et al., 2002) thereby enabling stabilization and performance studies. Furthermore, practitioners are familiar with energy concepts and therefore energy-based models may serve as a *lingua franca* amongst (control) engineers (Ortega et al., 2002). Two common energy-based models are provided by the Lagrangian (Chung et al., 2009) and port-Hamiltonian framework (Fujimoto, Sakurama, & Sugie, 2003; Shaik, Zonetti, Ortega, Scherpen, & van der Schaft, 2012).

In particular, the use of port-Hamiltonian (pH) systems has proven highly successful in many applications, see Duindam, Macchelli, and Stramigioli (2009), Ortega et al. (2002), Shaik et al. (2012) and references therein. In the port-Hamiltonian framework the plant and controller are viewed as energy processing dynamical systems and the models developed in this framework capture the energy, interconnection and dissipation structure of the system explicitly.

This work provides a theoretical framework for the spatial distribution of satellites on circular orbits. Preliminary versions of this work have appeared at (Vos, Scherpen, & van der Schaft, 2013; Vos, Shaik, Scherpen, & van der Schaft, 2013). The control system consists of virtual springs and dampers and builds upon (van der Schaft & Maschke, 2013) modeling the interaction between satellites in the constellation by a graph. Stability is proven using the energy function of the closed-loop system as a Lyapunov function candidate.

For satellite constellations the limited availability of propellant asks for energy-efficient control schemes. Insight into the controller's power requirements and energy consumption is inherent to the port-Hamiltonian framework making use of the energy functions.

We first recall some essential elements from port-Hamiltonian theory and graph theory, which are used throughout the remainder of this paper. Section 2 presents the dynamical model for the satellites and derive the error dynamics w.r.t. the circular target orbit. Section 3 then follows with the description of the control system. The main result is presented and analyzed in Section 4, followed by simulation results to illustrate the effectiveness of our approach in Section 5. Finally in Section 6 concluding remarks are presented including some directions for future work.

### 1.1. Preliminaries

The port-Hamiltonian (pH) framework is an energy-based framework which describes a large class of (nonlinear) systems including passive mechanical and electrical systems (Duindam et al., 2009). Define the state $x \in \mathbb{R}^n$, input $u \in \mathbb{R}^m$, and output $y \in \mathbb{R}^m$. The product of the input and output $u^T(t)y(t)$ equals the power supplied to the system. The general form of a pH system is given by

$$\begin{aligned} \dot{x} &= [J(x) - R(x)]\frac{\partial H}{\partial x}(x) + g(x)u \\ y &= g^T(x)\frac{\partial H}{\partial x}(x) \end{aligned} \tag{1}$$

where the structure matrix $J(x) \in \mathbb{R}^{n\times n}$ is skew-symmetric (i.e., $J(x) = -J(x)^T$), and the dissipation matrix $R(x) \in \mathbb{R}^{n\times n}$ is positive semi-definite (i.e., $R(x) = R(x)^T \geq 0$). The Hamiltonian $H(x) : \mathbb{R}^n \to \mathbb{R}$ equals the total energy stored in the system, and its time derivative is given by $\dot{H} \leq u^T(t)y(t)$. Hence the increase in the stored energy is always equal to or smaller than the power supplied through the power-port $(u, y)$. Therefore (1) is a passive system if $H(x)$ is bounded from below. See Duindam et al. (2009) for a concise overview of the port-Hamiltonian framework.

The pairwise interaction between satellites in the constellation is modeled by an undirected connected graph $\mathcal{G}(\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ denotes the set of $n$ nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ denotes the set of $m$ edges. Satellites $i$ and $j$ can interact if there is an edge $(i, j) \in \mathcal{E}$. Each edge has an orientation by assigning a positive sign to one end and a negative sign to the other end. The incidence matrix $B$ associated to $\mathcal{G}(\mathcal{V}, \mathcal{E})$ describes which nodes are coupled by an edge, and is defined as

$$b_{ik} = \begin{cases} +1 & \text{if node } i \text{ is the head node of the edge } k \\ -1 & \text{if node } i \text{ is the tail node of the edge } k \\ 0 & \text{otherwise.} \end{cases}$$

For more details on graphs see e.g. Bollobás (1998).

## 2. Port-Hamiltonian formulation of translational dynamics of the satellite constellation

Consider an inertial frame of reference in Cartesian coordinates, with the origin at the center of earth. The $z$-axis of the reference frame is assumed to be normal to the orbital plane of interest. We are merely interested in the satellite dynamics in the orbital plane, so we omit the dynamics along the $z$-axis. Each satellite is modeled as a point mass which is subject to the gravitational field of planet earth. Let $q_i^c = \left(q_{x,i}^c, q_{y,i}^c\right)^T$ and $p_i^c = \left(p_{x,i}^c, p_{y,i}^c\right)^T$ denote the position and the momentum in Cartesian coordinates of satellite $i$ in the inertial frame of reference (see Fig. 1). The port-Hamiltonian dynamics of satellite $i$ are given by

$$\begin{aligned} \dot{x}_i^c &= J_i^c \frac{\partial H_i^c}{\partial x_i^c}(x_i^c) + g_i^c f_i^c \\ v_i^c &= g_i^{cT} \frac{\partial H_i^c}{\partial x_i^c}(x_i^c), \end{aligned} \tag{2}$$

with state $x_i^c = \left(q_i^c, p_i^c\right)^T$, input force $f_i^c = \left(f_{x,i}^c, f_{y,i}^c\right)^T$, and output velocity $v_i^c = \left(v_{x,i}^c, v_{y,i}^c\right)^T$. The structure matrix $J_i^c$ and the input matrix $g_i^c$ are given by respectively $J_i^c = \begin{bmatrix} 0 & I_2 \\ -I_2 & 0 \end{bmatrix}$ and $g_i^c = \begin{bmatrix} 0 \\ I_2 \end{bmatrix}$. The Hamiltonian $H_i^c(x_i^c)$ is the sum of the kinetic energy (KE) of satellite $i$ and the gravitational potential energy (GPE), where earth is assumed to be a perfect sphere (i.e., deviations like the $J_2$-perturbation are neglected) (Alfriend, Srinivas, Gurfil, How, & Breger, 2010) and is given by

$$H_i^c(x_i^c) = \underbrace{\frac{1}{2m_i}(p_{x,i}^c)^2 + \frac{1}{2m_i}(p_{y,i}^c)^2}_{\text{KE}} - \underbrace{\frac{\mu_e m_i}{\left\|q_i^c\right\|}}_{\text{GPE}}, \tag{3}$$

with $m_i$ the mass of satellite $i$ and $\left\|q_i^c\right\|$ the distance w.r.t. the center of earth defined as $\left\|q_i^c\right\| = \sqrt{(q_{x,i}^c)^2 + (q_{y,i}^c)^2}$.

In contrast with most energy functions, Hamiltonian (3) is not bounded from below and therefore $H_i^c(x_i^c)$ cannot be directly used as a Lyapunov function candidate. However, from physics we have that $\left\|q_i^c\right\| > 0$ (see Assumption 1), and thus $H_i^c(x_i^c)$ is in fact bounded from below. Moreover, $H_i^c(x_i^c)$ has multiple critical points which might give rise to undesired equilibria. The critical points for $H_i^c(x_i^c)$ are defined as those $x_i^c$ for which $\frac{\partial H_i^c}{\partial x_i^c}(x_i^c) = 0$.

In order to facilitate the control design of equal distribution on a circular orbit we employ polar coordinates. Let $r_i \in \mathbb{R}$, $\phi_i \in [0, 2\pi]$, $p_i \in \mathbb{R}$, and $h_i \in \mathbb{R}$ denote the radial distance, azimuthal angle, momentum, and angular momentum of satellite $i$ in polar