Automatica 50 (2014) 1053–1065

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Cooperative semi-global robust output regulation for a class of nonlinear uncertain multi-agent systems[☆]

Youfeng Su[a], Jie Huang[b,1]

[a] College of Mathematics and Computer Science, Fuzhou University, Fuzhou, China

[b] Department of Mechanical and Automation Engineering, The Chinese University of Hong Kong, Shatin, N.T., Hong Kong

ARTICLE INFO

Article history:
Received 15 October 2012
Received in revised form
6 August 2013
Accepted 7 January 2014
Available online 26 February 2014

Keywords:
Cooperative control
Multi-agent systems
Output regulation
Internal model
Nonlinear uncertain systems

ABSTRACT

In this paper, we study the cooperative semi-global robust output regulation problem for a class of minimum phase nonlinear uncertain multi-agent systems. This problem is a generalization of the leader-following tracking problem in the sense that it further addresses such issues as disturbance rejection, robustness with respect to parameter uncertainties. To solve this problem, we first introduce a type of distributed internal model that converts the cooperative semi-global robust output regulation problem into a cooperative semi-global robust stabilization problem of the so-called augmented system. We then solve the semi-global stabilization problem via distributed dynamic output control law by utilizing and combining a block semi-global backstepping technique, a simultaneous high gain feedback control technique, and a distributed high gain observer technique.

1. Introduction

In the past decade, the leader-following tracking problem for various multi-agent systems such as, single-integrator systems (Jadbabaie, Lin, & Morse, 2003; Ren, 2007), double-integrator systems (Hu & Hong, 2007; Ren, 2008), general linear multi-agent systems (Li, Liu, Lin, & Ren, 2011), second order nonlinear systems with global Lipschitz assumption (Meng & Lin, 2012; Song, Cao, & Yu, 2010), thrust-propelled vehicles (Lee, 2012), and multiple mechanical systems (Dong, 2011), has been extensively studied, see also the recent books Qu (2009); Ren and Beard (2008). In particular, Dong (2011) employed the adaptive control technique to deal with some type of model uncertainty.

Recently, the cooperative output regulation problem for multi-agent systems has received more and more attention (Hong, Wang, & Jiang, 2013; Su & Huang, 2012, 2013; Wang, Hong, Huang, & Jiang, 2010; Yang, Stoorvogel, Grip, & Saberi, 2012). Instead of tracking a specific reference signal, this problem aims to design a distributed control law such that the output of each subsystem can asymptotically track a class of reference inputs in the presence of a class of disturbances and plant parameter uncertainties. Like the classical output regulation problem, here the class of reference inputs and the class of disturbances are both generated by a differential equation called the exosystem. Therefore, this problem can be viewed as a generalization of the leader-following tracking problem by treating the plant as the follower system and the exosystem as the leader system. So far, the problem has been widely studied for linear uncertain multi-agent systems in, say, Hong et al. (2013), Su and Huang (2013), Wang et al. (2010) and Yang et al. (2012). More recently, the cooperative robust output regulation problem for a class of nonlinear multi-agent systems in lower triangular form was further formulated and a global solution was obtained by a distributed state feedback control law in Su and Huang (2012). In the special case where the number of subsystems is equal to one, the problem in Su and Huang (2012) reduces to the conventional global robust output regulation problem as studied in Huang and Chen (2004).

However, like the conventional robust output regulation problem for a single nonlinear system in lower triangular form, the global solution cannot be obtained via an output feedback control law. Therefore, in this paper, we will further study a so-called cooperative semi-global robust output regulation problem for the class of uncertain nonlinear multi-agent systems to be described

[☆] This work has been supported in part by the Research Grants Council of the Hong Kong Special Administration Region under grant No. 412810, and in part by the National Natural Science Foundation of China under grant No. 61174049. The material in this paper was partially presented at the 2013 American Control Conference (ACC 2013), June 17–19, 2013, Washington, DC, USA. This paper was recommended for publication in revised form by Associate Editor Wei Ren under the direction of Editor Frank Allgöwer.

E-mail addresses: yfsu@fzu.edu.cn (Y. Su), jhuang@mae.cuhk.edu.hk (J. Huang).

[1] Tel.: +852 39438473; fax: +852 26036002.

http://dx.doi.org/10.1016/j.automatica.2014.02.010
0005-1098/

in (1). For the special case where the number of the subsystems of system (1) is equal to one, our problem reduces to the semi-global robust output regulation problem for a single nonlinear system (Khalil, 1994; Lan, Chen, & Huang, 2005; Serrani, Isidori, & Marconi, 2001).

In comparison with the semi-global robust output regulation problem for a single nonlinear system, our problem is technically more challenging in at least two ways. First, the system in Khalil (1994), Lan et al. (2005) and Serrani et al. (2001) is a single-input, single-output system. It can be converted into a semi-global stabilization problem of an augmented system through the employment of an internal model. The augmented system is still a single-input, single-output system whose stabilization problem can be handled by established techniques for the semi-global stabilization (Teel & Praly, 1995). In contrast, for a multi-agent system, the augmented system is a multi-input, multi-output nonlinear system and we have to develop specific techniques of block semi-global backstepping method that apply to multi-input, multi-output nonlinear systems. Second, due to the communication constraint which is described by a communication graph to be introduced in Section 2, we cannot use the full information of the system for feedback control, and we have to develop a distributed control law to stabilize the augmented system.

The rest of this paper is organized as follows: In Section 2, we give a precise description of cooperative semi-global robust output regulation problem. In Section 3, we introduce a type of distributed internal model that converts the cooperative semi-global robust output regulation problem into a cooperative semi-global robust stabilization problem of the augmented system. In Section 4, we present some technical lemmas that are applicable to the block lower triangular nonlinear systems. These lemmas can be viewed as block techniques of semi-global backstepping method. In Section 5, by utilizing these technical lemmas, a simultaneous high gain feedback control technique and a distributed version of high gain observer technique, we solve the semi-global stabilization problem of the augmented system via distributed dynamic output feedback control. Thus, our problem is solvable. In Section 5, we provide two examples to illustrate our design. Finally, in Section 6 we present our conclusions.

Notation: Given the column vectors $a_i,\ i = 1, \ldots, s$, we denote $\mathrm{col}(a_1, \ldots, a_s) = [a_1^T, \ldots, a_s^T]^T$. The compact set $\bar{Q}_R^s \triangleq \{x = \mathrm{col}(x_1, \ldots, x_s) \in \mathbb{R}^s : |x_i| \le R,\ i = 1, \ldots, s\}$. Given a positive definite and proper function $V : \mathbb{R}^s \to \mathbb{R}$, the symbol $\bar{\Omega}_c(V(x))$ denotes the compact set $\{x \in \mathbb{R}^s : V(x) \le c\}$, while the symbol $\Omega_c(V(x))$ denotes the open set $\{x \in \mathbb{R}^s : V(x) < c\}$. Given two sets $X_1 \in \mathbb{R}^{n_1}$ and $X_2 \in \mathbb{R}^{n_2}$, let $X_1 \times X_2 \triangleq \{\mathrm{col}(x_1, x_2) : x \in X_1, x_2 \in X_2\}$.

2. Problem statement

In this paper, we consider the following nonlinear uncertain multi-agent systems

$$\begin{aligned}
\dot{z}_i &= f_{0i}(z_i, x_{1i}, v, w),\\
\dot{x}_{si} &= x_{(s+1)i}, \quad s = 1, \ldots, r-1,\\
\dot{x}_{ri} &= f_{1i}(z_i, x_{1i}, \ldots, x_{ri}, v, w) + b_i(w)u_i,\\
y_i &= x_{1i}, \quad i = 1, \ldots, N,
\end{aligned} \tag{1}$$

where $z_i \in \mathbb{R}^{n_{z_i}}$, $x_i \triangleq \mathrm{col}(x_{1i}, \ldots, x_{ri}) \in \mathbb{R}^r$, $y_i, u_i, \in \mathbb{R}$, $v \in \mathbb{R}^q$, $w \in \mathbb{R}^{n_w}$ represents the parameter uncertainty. The exosystem is given by

$$\dot{v} = Sv, \qquad y_0 = q_0(v, w), \tag{2}$$

where $y_0 \in \mathbb{R}$ is the output of the exosystem. Then, for $i = 1, \ldots, N$, the regulated errors for the subsystems are defined as

$$e_i = y_i - y_0. \tag{3}$$

We assume the functions $f_{ki}(\cdot),\ b_i(\cdot),\ k = 0, 1,\ i = 1, \ldots, N$, and $q_0(\cdot)$ are sufficiently smooth functions with $f_{ki}(0, \ldots, 0, w) = 0,\ k = 0, 1$, and $q_0(0, w) = 0$.

The plant (1) and exosystem (2) together can be viewed as a multi-agent system of $N + 1$ agents with the exosystem as the leader and all the subsystems of (1) as the followers. With respect to (1) and (2), we can define a digraph[2] $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}\}$ where $\bar{\mathcal{V}} = \{0, 1, \ldots, N\}$ with the node 0 associated with the exosystem and the other N nodes associated with the N followers, respectively, and $(j, i) \in \bar{\mathcal{E}},\ j = 0, 1, \ldots, N$ and $i = 1, \ldots, N$, if and only if the control u_i can make use of $y_i - y_j$ for feedback control. Thus our control law is of the following form

$$\begin{aligned}
u_i &= u_i(\xi_i, y_i - y_j, j \in \mathcal{N}_i),\\
\dot{\xi}_i &= g_i(\xi_i, y_i - y_j, j \in \mathcal{N}_i),
\end{aligned} \tag{4}$$

where $\mathcal{N}_i = \{j : (j, i) \in \bar{\mathcal{E}}\}$, and u_i and g_i are sufficiently smooth functions vanishing at the origin, and $\xi_i \in \mathbb{R}^{n_i}$ with n_i to be defined later. A control law of the form (4) is called a distributed dynamic output feedback control law because the control of each subsystem can only take the output information of its neighbors and itself for feedback control. We call the composition of (1) and (4) as the overall closed-loop system which can be put in the following form

$$\dot{x}_c = f_c(x_c, v, w), \tag{5}$$

where $x_c = \mathrm{col}(z_1, x_1, \xi_1, \ldots, z_N, x_N, \xi_N) \in \mathbb{R}^{n_c}$ for some integer n_c and f_c is sufficiently smooth satisfying $f_c(0, 0, w) = 0$ for all $w \in \mathbb{R}^{n_w}$. Then our problem can be described as follows:

Given systems (1) and (2), the digraph $\bar{\mathcal{G}}$, a real number $R > 0$, and compact subsets $\mathbb{V}_0 \subseteq \mathbb{R}^q$ *and* $\mathbb{W} \subseteq \mathbb{R}^{n_w}$ *which contain the origins of the respective Euclidean spaces, find a control law of the form (4) such that for any $v(0) \in \mathbb{V}_0$, $w \in \mathbb{W}$, and $x_c(0) \in \bar{Q}_R^{n_c}$, the trajectory of the closed-loop system (5) starting from $x_c(0)$ and $v(0)$ exists and is bounded for all $t \ge 0$, and $\lim_{t\to\infty} e(t) = 0$, where $e = \mathrm{col}(e_1, \ldots, e_N)$.*

The above problem will be called *cooperative regional robust output regulation problem* for the nonlinear multi-agent system (1) on the compact subset $\bar{Q}_R^{n_c} \times \mathbb{V}_0 \times \mathbb{W}$. If for any $R > 0$, and any compact subsets $\mathbb{V}_0 \subseteq \mathbb{R}^q$ and $\mathbb{W} \subseteq \mathbb{R}^{n_w}$ which contain the origins of the respective Euclidean spaces, the cooperative regional robust output regulation problem for the nonlinear multi-agent system (1) on the compact subset $\bar{Q}_R^{n_c} \times \mathbb{V}_0 \times \mathbb{W}$ is solvable, then we say that the *cooperative semi-global robust output regulation problem* for the nonlinear multi-agent system (1) is solvable.

Remark 1. If the control u_i of each subsystem can access e_i for feedback control, then we can design N individual controllers of the form $u_i = u_i(\eta_i, e_i),\ \dot{\eta}_i = g_i(\eta_i, e_i)$ to solve the problem using the approach in, say, Lan et al. (2005) and Serrani et al. (2001). Such a control scheme is called (purely) decentralized control. However, due to the communication constraint among the agents, it is unrealistic to assume that all the followers know the information of the leader, i.e., the information of $e_i = y_i - y_0$. Thus, what makes our control law (4) interesting is that only those followers which are the neighbors of the leader need to know e_i. Other followers can only indirectly access e_i by sharing information with their neighbors. That is why we call our problem as cooperative semi-global robust output regulation problem for the multi-agent system (1).

[2] See Appendix A for a self-contained summary of digraph.