Automatica 50 (2014) 1217–1226

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

A low-complexity global approximation-free control scheme with prescribed performance for unknown pure feedback systems[☆]

Charalampos P. Bechlioulis[a], George A. Rovithakis[b,1]

[a] Control Systems Laboratory, School of Mechanical Engineering, National Technical University of Athens, 9 Heroon Polytechniou Str., Zografou, Athens 15780, Greece

[b] Department of Electrical & Computer Engineering, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece

ARTICLE INFO

Article history:
Received 20 November 2012
Received in revised form
7 January 2014
Accepted 22 January 2014
Available online 6 March 2014

Keywords:
Nonaffine systems
Uncertain systems
Global approximation-free control
Low-complexity
Prescribed performance

ABSTRACT

A universal, approximation-free state feedback control scheme is designed for unknown pure feedback systems, capable of guaranteeing, for any initial system condition, output tracking with prescribed performance and bounded closed loop signals. By prescribed performance, it is meant that the output error converges to a predefined arbitrarily small residual set, with convergence rate no less than a certain prespecified value, having maximum overshoot less than a preassigned level. The proposed state feedback controller isolates the aforementioned output performance characteristics from control gains selection and exhibits strong robustness against model uncertainties, while completely avoiding the explosion of complexity issue raised by backstepping-like approaches that are typically employed to the control of pure feedback systems. In this respect, a low complexity design is achieved. Moreover, the controllability assumptions reported in the relevant literature are further relaxed, thus enlarging the class of pure feedback systems that can be considered. Finally, simulation studies clarify and verify the approach.

1. Introduction

During the past several years, controlling systems with complex and uncertain nonlinear dynamics has attracted considerable research effort. Significant progress has been achieved through adaptive feedback linearization (Sastry & Isidori, 1989), adaptive backstepping (Krstic, Kanellakopoulos, & Kokotovic, 1995) and adaptive neural network/fuzzy logic control (Farrell & Polycarpou, 2006; Ge, Hang, Lee, & Zhang, 2002; Lewis, Jagannathan, & Yesildirek, 1999; Rovithakis & Christodoulou, 2000; Spooner, Maggiore, Ordonez, & Passino, 2002). The aforementioned results were obtained for systems in affine form, that is, for plants linear in the control input variables. However, there exist applications such as chemical processes and flight control systems, which cannot be expressed in an affine form. The difficulty associated with the control design of such systems arises from the fact that an explicit inverting control design is, in general, impossible, even though the inverse exists. Initially, nonaffine systems in low triangular canonical form (i.e., system nonlinearities satisfy a matching condition) were considered (Ge & Zhang, 2003; Hovakimyan, Lavretsky, & Cao, 2008; Hovakimyan, Nardi, & Calise, 2002; Labiod & Guerra, 2007; Leu, Wang, & Lee, 2005; Park, Huh, Kim, Seo, & Park, 2005; Park, Park, Kim, & Moon, 2005; Wang, Chien, & Lee, 2011; Yang & Calise, 2007; Zhao & Farrell, 2007). Subsequently, as the problem became more apparent, the significantly more complex as well as more general class of pure feedback nonaffine systems (i.e., all system states and control inputs appear implicitly in the system nonlinearities) was tackled (Chien, Wang, Leu, & Lee, 2011; Ge & Wang, 2002; Ren, Ge, Su, & Lee, 2009; Wang, Chien, Leu, & Lee, 2010; Wang, Ge, & Hong, 2010; Wang, Hill, Ge, & Chen, 2006; Wang & Huang, 2002; Wang, Liu, & Shi, 2011; Zhang & Ge, 2008; Zhang, Wen, & Zhu, 2010; Zhang, Zhu, & Yang, 2012; Zou, Hou, & Tan, 2008). More specifically, in case of single-input single-output nonaffine systems with unknown nonlinearities, fuzzy systems and neural networks have been utilized to approximate an ‘ideal controller’, whose existence is guaranteed by the Implicit Function Theorem. Works incorporating the Mean Value Theorem (Chien et al., 2011; Ge & Wang, 2002; Ge & Zhang, 2003; Labiod & Guerra, 2007; Ren et al., 2009; Wang, Chien et al., 2011, 2010; Wang, Ge et al., 2010; Wang et al.,

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Antonio Loria under the direction of Editor Andrew R. Teel.

E-mail addresses: chmpechl@mail.ntua.gr (C.P. Bechlioulis), robi@eng.auth.gr (G.A. Rovithakis).

[1] Tel.: +30 2310995820; fax: +30 2310996124.

http://dx.doi.org/10.1016/j.automatica.2014.02.020
0005-1098/

2006; Wang, Liu et al., 2011; Zhang & Ge, 2008; Zhang et al., 2010, 2012), the Taylor series expansion (Leu et al., 2005), and the contraction mapping method (Park, Huh et al., 2005; Park, Park et al., 2005) have been proposed to decompose the original nonaffine system into an affine in the control part and a nonaffine part representing generalized modeling errors. Subsequently, standard robust adaptive control tools were employed. However, approximating this "ideal controller" is a difficult task, leading also to complex neural network and fuzzy system structures. In Hovakimyan et al. (2002), Yang and Calise (2007), instead of seeking a direct solution to the inverse problem, an analytically invertible model was introduced and a neural network was designed to compensate for the inversion error. Moreover, in Hovakimyan et al. (2008), singular perturbation theory was applied to derive an adaptive dynamical inversion method for uncertain nonaffine systems.

Despite the recent progress in the control of unknown nonaffine systems, certain issues still remain open. In fact, all aforementioned works have resorted to approximation-based techniques (i.e., neural networks and fuzzy systems) to deal with the model uncertainties of the system. Unfortunately, this approach inherently introduces certain issues affecting closed loop stability and robustness. Specifically, even though the existence of a closed loop initialization set as well as of control gain values that guarantee closed loop stability is proven, the problem of proposing an explicit constructive methodology capable of a priori imposing the required stability properties is not addressed. As a consequence, the produced control schemes yield inevitably reduced levels of robustness against modeling imperfections. Moreover, the results are restricted to be local as they are valid only within the compact set where the capabilities of the universal approximators hold. Furthermore, the introduction of approximating structures increases the complexity of the proposed control schemes in the sense that extra adaptive parameters have to be updated (i.e., nonlinear differential equations have to be solved numerically) and extra calculations have to be conducted to output the control signal, thus making implementation difficult. Additionally, all aforementioned works guarantee convergence of the tracking error to a residual set, whose size depends on explicit design parameters and some unknown bounded terms. However, no systematic procedure exists to accurately compute the required upper bounds, thus making the a priori selection of the design parameters to satisfy certain steady state behavior practically impossible. Moreover, transient performance related to overshoot and convergence rate is difficult to be established even in the case of known nonlinearities. An approach to tackle this problem is in terms of the $\mathcal{L}_2$ norm of the tracking error that is derived to be a function of explicit design parameters and initial estimation errors (Ge & Zhang, 2003; Hovakimyan et al., 2008, 2002; Ren et al., 2009; Wang et al., 2006). However, the aforementioned performance index is connected only indirectly with the actual system response. Therefore, a reduction of the $\mathcal{L}_2$ norm of the tracking error results in an overall transient performance improvement, with, on the other hand, no specific connection to trajectory-oriented metrics such as overshoot and convergence rate. Thus, the problem of guaranteeing prescribed performance for nonaffine systems still remains. By prescribed performance, it is meant that the tracking error converges to a predefined arbitrarily small residual set, with convergence rate no less than a prespecified value, exhibiting maximum overshoot less than a preassigned level. In this direction, the problem was originally posed and solved in Miller and Davison (1991) for a sufficiently general class of LTI systems. Extensions to the nonlinear paradigm were first made possible via the so-called funnel control approach, first appeared in Ilchmann, Ryan, and Sangwin (2002). According to Ilchmann and Ryan (2008), funnel control is a continuation of the adaptive high-gain control methodology with the advancement of replacing the monotonically increasing control gain in the former by a time-varying function which admits high values when the output error is close to the funnel boundary, resulting in a nonlinear and time-varying proportional control scheme of significant simplicity for classes of nonlinear systems having known relative degree one (Hopfe, Ilchmann, & Ryan, 2010; Ilchmann, Logemann, & Ryan, 2010; Ilchmann & Ryan, 2009; Ilchmann et al., 2002; Ilchmann, Ryan, & Trenn, 2005) and lately for relative degree two (Hackl, Hopfe, Ilchmann, Mueller, & Trenn, 2013). Handling the problems introduced by higher known relative degree for a class of nonlinear systems, via the output feedback funnel control methodology, is reported in Ilchmann, Ryan, and Townsend (2006), Ilchmann, Ryan, and Townsend (2007). However, quoting Ilchmann and Ryan (2008) (Section 6, p.122) "a backstepping procedure is used which complicates the feedback structure". Therefore, the problem of constructing a controller of simplicity comparable to the relative degree one case, but for nonlinear systems of higher known relative degree, is still an open issue within the funnel control framework.

Working independently, Bechlioulis and Rovithakis proposed an alternative approach, named Prescribed Performance Control (PPC), to succeed the same control objective. Utilizing a transformation function that incorporates the desirable performance characteristics, PPC suggests transforming the original controlled system into a new one. Guaranteeing the uniform boundedness of the states of the latter, through proper control action, proves necessary and sufficient to solve the problem for the former. PPC methodology has been employed to design neuro-adaptive controllers for various classes of nonlinear systems having known high relative degree, namely feedback linearizable (Bechlioulis & Rovithakis, 2008), strict feedback (Bechlioulis & Rovithakis, 2009) and general MIMO affine in the control (Bechlioulis & Rovithakis, 2010). In Bechlioulis and Rovithakis (2011) the use of neural network approximators has been further relaxed and a universal controller following the PPC methodology is designed for general SISO strict feedback systems of known high relative degree that involve besides unknown nonlinearities, unknown dynamic uncertainties as well, avoiding the use of backstepping and of filtering; thus resulting in a low complexity design. It is also shown in Bechlioulis and Rovithakis (2011) that the results can be easily extended to cover the presence of bounded state measurement errors as well as to MIMO nonlinear systems in block triangular form.

In this work, under the assumption of full state availability, the results of Bechlioulis and Rovithakis (2011) are extended to the problem of controlling, with prescribed performance, unknown pure feedback systems of known high relative degree; keeping however the complexity of the control solution at low levels. In this direction, an approximation-free state feedback control scheme is proposed that achieves global results in the sense that given any initial system condition and any output performance specifications, regarding the steady state error, the convergence rate and the overshoot, the control objective is satisfied with bounded closed loop signals. Furthermore, the output performance is isolated from control gains selection and the robustness against model uncertainties is greatly extended. In fact, any system in pure feedback form obeying certain controllability assumptions can be controlled by the proposed scheme without altering the controller structure or the control gain values. Moreover, only the desired trajectory and none of its higher order derivatives is required, as opposed to existing control algorithms for pure feedback systems. The proposed scheme does not incorporate any prior knowledge of system nonlinearities or even of some corresponding upper/lower bounding functions, relaxing thus significantly the key assumptions made in the related literature. Compared with works residing in backstepping-like approaches to handle the issue of known high relative degree (including funnel control) that may incorporate output feedback only, the proposed methodology completely