Automatica 50 (2014) 2366–2372

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Adaptive multi-agent containment control with multiple parametric uncertain leaders[☆]

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ARTICLE INFO

Article history:
Received 3 May 2013
Received in revised form
8 December 2013
Accepted 29 April 2014
Available online 13 August 2014

Keywords:
Multi-agent systems
Multiple leaders
Containment
Uncertain parameter
Adaptive internal model

ABSTRACT

In this paper, an adaptive containment control is considered for a class of multi-agent systems with multiple leaders containing parametric uncertainties. The agents are heterogeneous though their dynamics have the same relative degree and are minimum phase, while the interconnection topology is described by a general directed graph. A distributed containment control is proposed for the agents to enter the moving convex set spanned by the leaders, based on an adaptive internal model and a recursive stabilization control law.

1. Introduction

Recent decades have witnessed a lot of research interest in the coordination of multi-agent systems, especially the leader-following coordination of linear systems (Hong, Wang, & Jiang, 2013; Li, Liu, Ren, & Xie, 2013). Distributed output regulation has been studied as a framework for such problems when the leader's dynamics are totally different from those of the followers, and distributed control based on internal model (IM) is shown to be an effective way to track a moving leader (Hong et al., 2013; Su & Huang, 2013; Wang, Hong, Huang, & Jiang, 2010). In fact, IM has been widely used in conventional output regulation (Huang, 2004; Isidori, Marconi, & Praly, 2012). Moreover, adaptive IMs were proposed for the output regulation with uncertain exosystems (Liu, Chen, & Huang, 2009; Obregón-Pulido, Castillo-Toledo, & Loukianov, 2011; Serrani, Isidori, & Marconi, 2001), and then distributed output regulation with an uncertain leader was studied for multi-agent systems (Su & Huang, 2013).

Also, multi-agent containment control attracts more and more attention by forcing follower agents to enter a given convex set (maybe spanned by a group of leaders) in a distributed way. Various containment controllers were designed for fixed or switched interconnection topologies and related connectivity conditions for containment were discussed (e.g., Cao, Stuart, Ren, & Meng, 2011; Lou & Hong, 2012; Meng, Ren, & You, 2010; Shi & Hong, 2009; Shi, Hong, & Johansson, 2012).

The objective of our paper is to study containment control with multiple leaders containing parametric uncertainties. Our contribution can be summarized as follows:

(i) The leader model we considered is different from those in existing containment results for the leaders with known dynamics (Cao et al., 2011; Mei, Ren, & Ma, 2012; Meng et al., 2010), though the parametric uncertainties may exist in the followers' dynamics (see Mei et al., 2012). For the leaders with uncertain parameters, the existing containment methods cannot be applied directly. To deal with this hard problem (noting that adaptive control design is essentially nonlinear), we propose an adaptive IM-based design for containment control.

(ii) Even when there is one leader, our problem is different from the leader-following problem with the leader containing uncertain inputs (e.g., Li et al. (2013), where the leader shares the same dynamics with the followers and sends the full state information to the agents connected to it). In this paper, the leaders have uncertain parameters and their dynamics are different from those of the followers. Moreover, the containment problem is solved using agents' relative outputs.

(iii) The results are different from those given for distributed output regulation when there is one leader with parametric

☆ This work is supported by National Natural Science Foundation of China under Grants 61174071 and 61273090. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Tamas Keviczky under the direction of Editor Frank Allgöwer.

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http://dx.doi.org/10.1016/j.automatica.2014.07.019
0005-1098/

uncertainties (Su & Huang, 2013). The follower agents with relative degree one and an undirected interconnection topology among followers were considered in Su and Huang (2013). Here we develop an adaptive IM-based approach to study the leader-following/containment when the interconnection topology is a general digraph and the followers are heterogeneous with relative degree greater than one.

2. Formulation

In this paper, we consider a multi-agent system composed of N follower agents and K leaders. The followers' dynamics are described as follows:

$$\begin{aligned}\dot{\check{x}}_i &= \bar{A}_i(\sigma)\check{x}_i + \bar{B}_i(\sigma)u_i + \sum_{j=1}^{K} \bar{E}_{ij}(\sigma)v_j \\ y_i &= \bar{C}_i(\sigma)\check{x}_i, \quad i = 1, \ldots, N \end{aligned} \tag{1}$$

where $\check{x}_i \in \mathbb{R}^{n_i}$, $u_i, y_i \in \mathbb{R}$, and $\sigma \in \mathbb{R}^{n_\sigma}$ denotes the uncertain parameter in the agents' dynamics. The leaders are described as

$$\dot{v}_j = S_j(\omega)v_j, \quad j = 1, \ldots, K \tag{2}$$

where $v_j \in \mathbb{R}^{n_v}$ and $\omega \in \mathbb{W} \subset \mathbb{R}^{n_\omega}$ represents the uncertain parameter in a fixed compact set $\mathbb{W}$. $y_{N+j} = F_j v_j \in \mathbb{R}$ is the output of leader j. It is assumed that the initial conditions of the leaders belong to a fixed compact set $\mathbb{V} \subset \mathbb{R}^{Kn_v}$, i.e., $(v_1(0), \ldots, v_K(0)) \in \mathbb{V}$.

Remark 2.1. There are two different types of parametric uncertainties in our problem: uncertainty σ in the followers, to be handled with robust technique (see the distributed stabilization control in Theorem 4.1), and uncertainty ω in the leaders, to be tackled using adaptive internal model. In what follows, we use $\bar{A}_i, \bar{B}_i, \bar{C}_i, \bar{E}_{ij}$ by dropping σ for simplicity.

With regarding the followers and leaders as nodes, a digraph (directed graph) $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with a node set $\mathcal{V} = \{1, \ldots, N+K\}$ and an arc set $\mathcal{E}$ can be defined to describe the interconnection for the multi-agent system (1) with the leaders (2), where the first N nodes are associated with the N followers of system (1) and the last K nodes without any neighbors represent the leaders of (2). The arc set $\mathcal{E}$ contains an arc, denoted by (i, j), if node j can get the output y_i of node i. The set $\mathcal{N}_i = \{j \in \mathcal{V} : (j, i) \in \mathcal{E}\}$ denotes the set of neighbors of node i. A *path* from i to j in $\mathcal{G}$ is a node/arc sequence $i_1 e_1 i_2 e_2 \cdots e_{m-1} i_m$ of distinct nodes i_κ and arcs $e_\kappa = (i_\kappa, i_{\kappa+1}) \in \mathcal{E}$ for $\kappa = 1, 2, \ldots, m-1$ with $i_1 = i,\ i_m = j$. The adjacency matrix of $\mathcal{G}$ is denoted as $\mathcal{A} = (a_{ij}) \in \mathbb{R}^{(N+K)\times(N+K)}$, where $a_{ii} = 0$, $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian of $\mathcal{G}$ is denoted by $\mathcal{L} = (l_{ij}) \in \mathbb{R}^{(N+K)\times(N+K)}$ with $l_{ii} = \sum_{j=1}^{N+K} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j, i, j = 1, \ldots, N+K$ (Godsil & Royle, 2001).

Convex sets are used to describe containment problems. A set $\mathcal{S} \subset \mathbb{R}^m$ is convex if $(1-\gamma)x + \gamma y \in \mathcal{S}$ whenever $x, y \in \mathcal{S}$ and $0 < \gamma < 1$ (Rockafellar, 1972). The intersection of all convex sets containing $\mathcal{S}$ is the convex hull of $\mathcal{S}$, denoted by $co(\mathcal{S})$. The convex hull of a set of points $y_1, \ldots, y_K \in \mathbb{R}^m$ is a polytope, denoted by $co\{y_1, \ldots, y_K\}$. Denote $\mathcal{B}_\rho^x \triangleq \{x \in \mathbb{R}^n : \|x\| \le \rho\}$ for a constant $\rho > 0$, and $\Omega_c(W) \triangleq \{x \in \mathbb{R}^n : W(x) \le c\}$ for a constant $c > 0$ and a smooth positive definite function $W(x)$. For a closed convex set $\mathcal{S}$, denote $\|x\|_{\mathcal{S}} \triangleq \inf\{\|x - y\| : y \in \mathcal{S}\}$, where $\|\cdot\|$ denotes the Euclidean norm.

Define a relative neighbor-based output e_{iv}:

$$e_{iv} = \frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}(y_i - y_j), \quad i = 1, \ldots, N \tag{3}$$

where $|\mathcal{N}_i|$ is the cardinality of the set $\mathcal{N}_i$.

Definition 2.1. The (semi-global) adaptive containment problem is solved for the multi-agent system (1) with uncertain leaders (2) if, for any set $\mathcal{B}_\rho^{\check{x}}$ with $\check{x} = (\check{x}_1, \ldots, \check{x}_N)^T$,[2] we can find a distributed control

$$\dot{\varsigma}_i = g_i(e_{iv}, \varsigma_i, \varsigma_j, j \in \mathcal{N}_i), \qquad u_i = f_i(e_{iv}, \varsigma_i, \varsigma_j, j \in \mathcal{N}_i) \tag{4}$$

with $\mathcal{B}_{\rho'}^{\varsigma}$, and $\varsigma = (\varsigma_1, \ldots, \varsigma_N)$, $\varsigma_i \in \mathbb{R}^{n_{\varsigma i}}$ such that, for any $(\check{x}(0), \varsigma(0)) \in \mathcal{B}_\rho^{\check{x}} \times \mathcal{B}_{\rho'}^{\varsigma}$ and $(v_1(0), \ldots, v_K(0), \omega) \in \mathbb{V}\times\mathbb{W}$, the solution of the closed-loop system is bounded over $[0, \infty)$, and

$$\lim_{t\to\infty} \|y_i(t)\|_{\mathcal{C}(t)} = 0, \quad i = 1, \ldots, N$$

where $\mathcal{C}(t) = co\{y_{N+1}(t), \ldots, y_{N+K}(t)\}$.

To solve the problem, standard assumptions are listed.

Assumption 2.1. For each $j = 1, \ldots, K$, all the eigenvalues of $S_j(\omega)$ are distinct with zero real parts for all ω.

Assumption 2.1 is widely used in the study of output regulation with exosystem/leader containing uncertain parameters. Under this assumption, for any $(v_1(0), \ldots, v_K(0), \omega) \in \mathbb{V}\times\mathbb{W}$, there is a known compact set $\Sigma \subset \mathbb{R}^{Kn_v+n_\omega}$ such that $d(t) \triangleq (v_1(t), \ldots, v_K(t), \omega) \in \Sigma$ for $t \ge 0$. With uncertainty ω, the leaders may produce sinusoidal signals with arbitrary unknown frequencies, amplitudes and phases.

Assumption 2.2. For $i = 1, \ldots, N$, the system (1) has the same relative degree r (that is, $\bar{C}_i\bar{A}_i^k\bar{B}_i = 0$, $k = 0, 1, \ldots, r-2$, and $\bar{C}_i\bar{A}_i^{r-1}\bar{B}_i \neq 0$) with a known high-gain frequency $\bar{C}_i\bar{A}_i^{r-1}\bar{B}_i$, and is minimum phase.

Without loss of generality, we assume $\bar{C}_i\bar{A}_i^{r-1}\bar{B}_i = 1$ and $r \ge 2$.

Assumption 2.3. For each follower, there exists at least one leader that has a directed path to it.

Remark 2.2. Under Assumption 2.3, the Laplacian $\mathcal{L}$ of the digraph $\mathcal{G}$ can be expressed as

$$\mathcal{L} = \left[\begin{array}{c|c} H_1 & H_2 \\ \hline 0_{K\times N} & 0_{K\times K}\end{array}\right], \quad H_1 \in \mathbb{R}^{N\times N},\ H_2 \in \mathbb{R}^{N\times K}.$$

From Meng et al. (2010), all the eigenvalues of H_1 have positive real parts; each entry of $-H_1^{-1}H_2$ is nonnegative; and each row of $-H_1^{-1}H_2$ has a sum equal to 1. By Theorem 2.3 of Berman and Plemmons (1979) (p. 134), there is a matrix $Q = diag(q_1, \ldots, q_N)$, $q_i > 0$ such that $QH_1 + H_1^TQ$ is positive definite. Let λ_0 be the minimum eigenvalue of $QH_1 + H_1^TQ$.

3. Preliminary results

To solve the containment problem, we provide preliminary results in this section. At first we convert the containment problem into a leader-following problem with a virtual leader. Define the tracking error for each agent

$$e_i = y_i + (H_1^{-1}H_2)^{(i)}Fv, \quad i = 1, \ldots, N \tag{5}$$

where $(H_1^{-1}H_2)^{(i)}$ denotes the ith row of the matrix $H_1^{-1}H_2$, $F = block\,diag(F_1, \ldots, F_K)$, and $v = (v_1, \ldots, v_K)^T$. Thus, we can define a multi-agent system with a virtual leader as follows.

$$\dot{\check{x}}_i = \bar{A}_i\check{x}_i + \bar{B}_iu_i + \bar{E}_iv, \qquad y_i = \bar{C}_i\check{x}_i, \quad i = 1, \ldots, N \tag{6}$$

where $\bar{E}_i = block\,diag(\bar{E}_{i1}, \ldots, \bar{E}_{iK})$, and

$$\dot{v} = \bar{S}(\omega)v, \qquad \bar{S}(\omega) = block\,diag(S_1(\omega), \ldots, S_K(\omega)). \tag{7}$$

Lemma 3.1. *Under Assumption 2.3, suppose that the leader-following problem of (6) with the virtual leader (7) can be solved in the sense that, for any set $\mathcal{B}_\rho^{\check{x}}$, there is a distributed control (4) with $\mathcal{B}_{\rho'}^{\varsigma}$, such*

[2] For vectors $x_1, \ldots, x_n$, $(x_1, \ldots, x_n)^T \triangleq [x_1^T, \ldots, x_n^T]^T$.