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Guaranteed performance consensus in second-order multi-agent systems with hybrid impulsive control[☆]

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ABSTRACT

This paper studies the problem of guaranteed performance consensus in second-order multi-agent systems. Taking advantage of impulsive control, a hybrid cooperative control is presented, and an index function is introduced to assess the performance of agents. It is shown that by synthesizing the coupling weights and the average impulsive intermittence, multi-agent systems can achieve guaranteed performance consensus. A numerical example is given to illustrate the theoretical results.

1. Introduction

Much research has been devoted to decentralized coordination control of multi-agent systems (MASs) due to its broad applications, see, e.g., Guan, Liu, Feng, and Jian (2012); Li, Fu, Xie, and Zhang (2011); Olfati-Saber and Murray (2004); Qin and Gao (2012); Ren and Beard (2005); Yu, Chen, Cao, and Kurths (2010); Yu, Zheng, Chen, and Cao (2011). Moreover, with the development of control theory, guaranteed cost control (Chen, Wang, Li, & Lu, 2010; Xu, Teo, & Liu, 2008; Zhang, Wang, & Liu, 2008) has been a wider subject of much practical application. Enlightened by this "guaranteed cost" idea, we focus on the guaranteed performance consensus of second-order MASs, which means agents reach consensus under some performance constraint.

This paper adopts a hybrid impulsive control. This impulsive control could be meaningful for many practical applications while the operating time of the controller is much smaller than the sampling period, referring to Guan, Hill, and Shen (2005); Guan et al. (2012); Lu, Ho, and Cao (2010); Meng and Chen (2012). Knowing that most of the existing literature concentrate on the standard consensus of MASs with double/single-integrator dynamics, see Guan et al. (2012); Li et al. (2011); Olfati-Saber and Murray (2004); Qin and Gao (2012); Ren and Beard (2005); Yu et al. (2011) and references therein. Very few works focus on the assessment of performance within MASs while achieving consensus. In real world however, it is quite desirable to impose an adequate level of position performance on the process of achieving consensus.

2. Preliminaries and problem formulation

Let $\mathtt{G} = \{\mathtt{V}, \mathtt{E}, A\}$ be a directed graph with vertex set $\mathtt{V} = \{1, 2, \ldots, M\}$, edge set $\mathtt{E} \subset \mathtt{V} \times \mathtt{V}$, and a 0-1 adjacency matrix $A = (a_{ij})_{M\times M}$. $(j, i) \in \mathtt{E}$ denotes a communication channel from agent j to agent i directly, and vice versa. The neighboring set of agent i is $N_i = \{j|(j, i) \in \mathtt{E}\}$. The adjacency elements are nonnegative. For $i, j \in \mathtt{V}, j \in N_i \Leftrightarrow a_{ij} > 0$, and assume that $a_{ii} = 0, i \in \mathtt{V}$. The Laplacian matrix $L = (l_{ij})_{M\times M}$ is defined as $L = D - A$, where $D = diag(a_1, a_2, \ldots, a_M)$ with $a_i = \sum_{j=1}^{M} a_{ij}, i = 1, 2, \ldots, M$.

Lemma 1 (Yu et al. (2010)). *Assume that L is irreducible, then $L1_M = 0$, and there exists a positive vector $w = (w_1, w_2, \ldots, w_M)^T$ satisfying $\sum_{j=1}^{M} w_j = 1$, such that $w^T L = 0$. Let $\hat{W} = diag(w)$, $\hat{L} = \frac{\hat{W}L + L^T\hat{W}}{2}$ is positive semi-definite with a single eigenvalue 0.*

Consider a second-order MAS, its communication topology is $\mathtt{G} = \{\mathtt{V}, \mathtt{E}, A\}$, which is assumed to be connected. The dynamics of

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each agent is described as

$$\dot{x}^i(t) = v^i(t), \quad \dot{v}^i(t) = u^i(t), \tag{1}$$

where $x^i(t) \in \Re$, $v^i(t) \in \Re$ are the position and velocity states of agent i, respectively, $u^i(t) \in \Re$ is the control law, $i = 1, 2, \ldots, M$, $t \geq 0$.

The problem to be solved in this paper is as follows:

(a) Consensus: what kind of control law is admissible?
(b) Guaranteed performance: subject to some admissible control, is it possible for agents to achieve consensus with guaranteed performance?

Then we decompose $u^i(t)$ into two parts as

$$\dot{x}^i(t) = v^i(t), \quad \dot{v}^i(t) = u^i_{con}(t) + u^i_{opt}(t). \tag{2}$$

Consider problem (a), we define an impulsive time sequence $\mu = \{t_k \mid 0 \leq t_0 < t_1 < \cdots < t_k < \cdots, \lim_{k\to\infty} t_k = \infty\}$. For $T \geq t \geq 0$, denote the number of impulse times on $(t, T]$ by $N_\mu(t, T)$. To describe the relationship precisely between the impulsive times $N_\mu(t, T)$ and the impulsive intermittence on $(t, T]$, based on Hespanha and Morse (1999); Lu et al. (2010), we introduce the concept of "average impulsive intermittence".

Definition 2. $T_a > 0$ is said to be the average impulsive intermittence on $(t, T]$, if there exist two positive numbers N_0 and N_1 such that $\frac{T-t}{T_a} - N_0 \leq N_\mu(t, T) \leq \frac{T-t}{T_a} + N_1$.

Based on Guan et al. (2005, 2012), an impulsive consensus protocol is given as

$$u^i_{con}(t) = \alpha \sum_{k=0}^{\infty} \left[\sum_{j\in N_i} a_{ij}(x^j(t) - x^i(t)) + \sum_{j\in N_i} a_{ij}(v^j(t) - v^i(t)) \right] \delta(t - t_k), \tag{3}$$

where $\delta(\cdot)$ is the Dirac impulse, $\alpha > 0$ is the coupling strength and $i = 1, 2, \ldots, M$.

Remark 3. Compared with the standard consensus laws (Olfati-Saber & Murray, 2004; Qin & Gao, 2012; Yu et al., 2010, 2011), this impulsive controller has advantages of smaller control cost (only operating at sampling instants), less information (only exchanging at sampling instants), and simpler implementation (referring to the unit impulse signal).

Denote $\tilde{x}(t) = \sum_{j=1}^{M} w_j x^j(t)$ and $\tilde{v}(t) = \sum_{j=1}^{M} w_j v^j(t)$ as the weighted average position and velocity, respectively. Then the relative errors are $e^i_x(t) = x^i(t) - \sum_{j=1}^{M} w_j x^j(t)$ and $e^i_v(t) = v^i(t) - \sum_{j=1}^{M} w_j v^j(t)$, $i = 1, 2, \ldots, M$. For question (b), an index function associated with MAS (2) is thus defined as

$$J = \sum_{k=0}^{\infty} \int_{t_k}^{t_{k+1}} \left[e_x{}^T(s) P e_x(s) \right] ds, \tag{4}$$

where $e_x(t) = \left(e^1_x(t), \ldots, e^M_x(t)\right)^T$, $e_v(t) = \left(e^1_v(t), \ldots, e^M_v(t)\right)^T$, $P = diag(p_1, \ldots, p_M) > 0$ is the coupling matrix.

By the optimal control theory, $u^i_{opt}(t)$ is given as

$$u^i_{opt}(t) = -\beta v^i(t), \quad i = 1, 2, \ldots, M, \tag{5}$$

where $\beta > 0$ is the feedback gain.

Remark 4. Note that by seeking an upper bound of J, one can claim that agents achieve consensus with guaranteed position performance.

3. Main results

This section establishes conditions for MAS (2) to achieve guaranteed performance consensus under the hybrid control (3) and (5).

Definition 5. Guaranteed performance consensus of MAS (2) is said to be achieved, if for any initial states, $\lim_{t\to\infty} x^i(t) - x^j(t) = 0$, $\lim_{t\to\infty} v^i(t) = 0$, $i, j = 1, 2, \ldots, M$, and there exists $J^* > 0$ such that $J \leq J^*$.

Subject to the hybrid control (3) and (5), an equivalent error system of MAS (2) is: $\forall t \in (t_k, t_{k+1}]$,

$$\begin{cases} \begin{bmatrix} \dot{e}_x(t) \\ \dot{e}_v(t) \end{bmatrix} = \begin{bmatrix} 0 & I_M \\ 0 & -\beta I_M \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_v(t) \end{bmatrix}, \\ \begin{bmatrix} \triangle e_x(t_k) \\ \triangle e_v(t_k) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -\alpha L & -\alpha L \end{bmatrix} \begin{bmatrix} e_x(t_k) \\ e_v(t_k) \end{bmatrix}, \end{cases} \tag{6}$$

where $\triangle e_x(t_k) = e_x(t_k^+) - e_x(t_k)$. Moreover, it follows

$$e_x{}^T(t)(ww^T)e_x(t) = 0, \; e_v{}^T(t)(ww^T)e_v(t) = 0. \tag{7}$$

Given $0 < \varepsilon < 1$, define

$$\begin{cases} \Theta_1 = (1+\varepsilon)\alpha^2 L^T L, \\ \Theta_2 = (1+\frac{1}{\varepsilon})(I_M - \alpha L)^T (I_M - \alpha L). \end{cases} \tag{8}$$

Assumption 6. Given $\alpha > 0$, $0 < \varepsilon < 1$, there exist $\sigma_1, \sigma_2 > 0$, such that Θ_1, Θ_2 are positive semi-definite, and satisfy $\bar{\lambda} = \max\{\lambda_1, \lambda_2\} < 1$, where $\lambda_1 = \lambda_{\max}(\Theta_1 - \sigma_1 ww^T)$, $\lambda_2 = \lambda_{\max}(\Theta_2 - \sigma_2 ww^T)$.

Theorem 7. *Given $P = diag(p_1, \ldots, p_M) > 0$, if there exist α, $\beta > 0$, $0 < \varepsilon < \min\{2\beta, 1\}$ satisfying Assumption 6 and*

$$0 < T_a < \frac{\ln(\bar{\lambda})}{\frac{\varepsilon}{2} - 2\beta + \frac{2\beta}{1+N_1}}.$$

Then guaranteed performance consensus of MAS (2) is achieved under the hybrid control (3) and (5). An upper bound of J is given as

$$J \leq a e_x{}^T(0)\hat{L}e_x(0) + b e_v{}^T(0)e_v(0),$$

where $a = \frac{\sqrt{b\beta(b\varepsilon\lambda_1 - 2\lambda_2 \min\{p_i\})}}{\lambda_2 \lambda_{\max}(\hat{L})}$, $b \geq \frac{2\lambda_2}{\varepsilon\lambda_1}\max\{p_i\}$, T_a and N_1 are associated with the average impulsive intermittence of the impulse time sequence $\mu = \{t_k\}$.

Proof. Utilizing $\hat{L}$, w and $\hat{W}$ given in Lemma 1, a Lyapunov function candidate is constructed as

$$V(e_x(t), e_v(t)) = V_1(t) + V_2(t) = e_x(t)^T \hat{L} e_x(t) + e_v(t)^T e_v(t).$$

Via the general algebraic connectivity $a(L)$ defined in Yu et al. (2010) and $w^T e_x(t) = 0$, one has

$$V(t) \geq a(L) e_x{}^T(t)\hat{W}e_x(t) + e_v{}^T(t)e_v(t) = \begin{bmatrix} e_x{}^T(t) & e_v{}^T(t) \end{bmatrix} \begin{bmatrix} a(L)\hat{W} & 0 \\ 0 & I_M \end{bmatrix} \begin{bmatrix} e_x(t) \\ e_v(t) \end{bmatrix}. \tag{9}$$

Then with $\hat{W} > 0$, it follows $V(t) \geq 0$, and $V(t) = 0$ if and only if $e_x(t) = e_v(t) = 0$.

The total derivative of $V_2(t)$ with respect to (6) is

$$\dot{V}_2(t)\mid_{(6)} = -2\beta V_2(t), \; t \in (t_k, t_{k+1}],$$

which yields $V_2(t) = e^{-2\beta(t-t_k)} V_2(t_k^+)$.

By the inequality $c^T d + d^T c \leq \varepsilon c^T c + \frac{1}{\varepsilon} d^T d$, one has

$$V_2(t_k^+)\mid_{(6)} \leq e_x{}^T(t_k)\Theta_1 e_x(t_k) + e_v{}^T(t_k)\Theta_2 e_v(t_k),$$

where Θ_1 and Θ_2 are given by (8).