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Structural optimization under uncertainty in loading directions: Benchmark results

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ABSTRACT

In this paper, using a recently developed unified approach, benchmark results are presented for structural optimization when the only source of uncertainty is the variability of the applied load directions. The worst-load-direction oriented framework can be applied to a broad class of engineering optimization problems. In each case, the central element of the solution searching algorithm is a standard multi-load structure optimization problem, which using an appropriate method, can be solved within reasonable time. The varying load directions are handled by additional linear or nonlinear relations, which describe the allowable perturbations of the nominal load directions. The result of the optimization is a performance measure minimal design which is invariant to the investigated uncertainty type and satisfies the response constraints. In order to illustrate the viability and efficiency of the approach, problem-specific models, algorithms and detailed benchmark results are presented for volume minimization of 2D continuum structures with compliance constraints and weight minimization of 2D truss structures with displacement and stress constraints. In each case, the computational cost of the proposed approach is comparable with its fixed load direction oriented equivalent because the worst-load-direction identification process is searching on the space of allowable direction perturbations, which generally means an easier and smaller computational problem than the standard multi-load structure optimization.

1. Introduction

In the real-world structural optimization problems, the optimal performance obtained using conventional deterministic methods can be dramatically degraded in the presence of sources of uncertainty. The source of uncertainty may be the variability of applied loads, spatial positions of nodes, material properties, and so on [1,2]. Various approaches have been developed to account for different types of uncertainty in structural design and optimization methods. Generally speaking, these methods are mainly based on two kinds of uncertainty models: probabilistic (stochastic) or possibilistic (fuzzy) models. Based on stochastic uncertainty models of mechanical parameters, various techniques were proposed by Marti [3], Rozvany [4], Choi et al. [5], Lógó et al. [6], Lógó [7], Lógó et al. [8], and Lógó [9] for evaluation and estimation of failure probabilities that can be utilized in the reliability-based structural design methods. Based on the probability distribution of the random data, and using decision theoretical concepts, optimization problems under stochastic uncertainty are converted into appropriate deterministic substitute problems. Due to the occurring probabilities and expectations, approximate solution techniques must be applied. Several deterministic and stochastic approximation methods are provided by Marti [10]. A fuzzy optimization approach for geometrical nonlinear space trusses was presented by Kelesoglu and Ülker [11]. Assuming uncertain-but-bounded parameters Ben-Haim and Elishakoff [12] developed the so-called convex model, with which Pantelides and Ganzerli [13] proposed a robust truss optimization method. For various classes of convex optimization problems, a unified methodology of robust optimization was developed by Ben-Tal and Nemirovski [14]. Calafiore and El Ghaoui [15] proposed a method for finding the ellipsoidal bounds of the solution set of uncertain linear equations by using the semidefinite program (SDP), which was presented originally by [16]. It is important to mention the pioneer works of Achtziger et al. [17] and Achtziger [18] as well, related to truss topology design and topology optimization of discrete structures, which contain the basic theoretical backgrounds on the field of the truss topology optimization. The multiple-load truss topology and sizing optimization was presented first time by Achtziger [19]. Similar work was presented by Alvarez and Carrasco [20] for

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minimization of the expected compliance as an alternative approach for multiple load truss optimization, which seems also very useful to tackle the uncertain optimization problems. In a recent paper, Dunning et al. [21] introduced a new probabilistic approach for robust topology optimization to minimize the volume-constrained expected compliance with uncertainty in loading magnitude and applied direction, where uncertainties are assumed normally distributed and statistically independent. The presented model was formulated as a statistical model, which after some manipulation was replaced by an equivalent multi-load problem in the function of the number of perturbed loads. In another paper, Dunning et al. [22] presented a new model for simultaneous minimization of expectancy and variance of compliance in the presence of uncertainties in loading magnitude using exact formulations and analytically derived sensitivities. A new theoretical approach for truss optimization with uncertain load directions was presented by Csébfalvi [23]. Benchmark results for this worst-load-direction oriented approach were presented in Csébfalvi [24]. A worst-load-direction oriented approach for topology optimization of continuum structure with uncertain-but-bounded load directions was introduced by Csébfalvi [25]. Benchmark results for the proposed approach were presented in Csébfalvi [26].

In this paper, a framework is introduced for the worst-load-direction oriented structure optimization with uncertain loading directions which can be applied to a broad class of engineering optimization problems. The paper is organized as follows. Section 2 focuses on the mathematical formulation of the considered problem in the function of the investigated problem types. The model and a benchmark problem for sizing optimization of truss structures is presented in Section 3, where it will be shown that the result of optimization is a minimal-weight truss design, which is invariant to the investigated load uncertainty type and satisfies the displacement and stress constraints with the given constraint tolerance. The central element of the solution searching process is a standard multi-load sizing optimization model, in which a linear elastic material model used to describe the nonlinear relation between the design space and the response space. It will be shown that choosing an appropriate algorithm from the several possibilities, the nonlinear sizing optimization problem can be solved within reasonable time. The model and the benchmark problem of topology optimization of continuum structures with uncertain load directions is presented in Section 4, where it will be shown that the result is a volume-fraction-minimal design, which is invariant to the directional uncertainty and satisfies the compliance constraints defining the maximum allowed compliance values in the identified worst directions. In this paper, the standard “compliance minimization with given volume fraction” model was replaced with a newly developed reversed “volume fraction minimization with compliance constraints” model, where the compliance constraints define the maximal allowable compliance in the identified worst directions. In this study, the maximum allowed compliance is defined in the percentage of the nominal compliance. After this modification, the similarity between the two models is straightforward, because the standard truss optimization model can be described as “weight minimization with displacements and stress constraints”. In the reversed topology optimization model the objective function is linear and the standard nonlinear multi-load compliance computing algorithm developed for solid isotropic material (SIMP) used to define the constraints. The worst-load-direction identification process is searching on the space of allowable direction perturbations, which generally means an easier and smaller computational problem as the standard multi-load structure optimization. Finally, some concluding remarks are presented in Section 5.

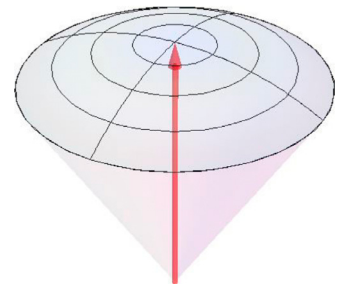

Fig. 1. A directionally uncertain unit load with maximum 30° allowable declination in any directions.

2. Structural optimization with varying load directions

In this paper a unified approach for structural optimization with varying load directions will be presented that can be applied to a broad class of engineering optimization problems. The investigation was motivated by the fact that the optimal structure designs are frequently sensitive to the directional uncertainties of the applied loads. Hence a structure engineer must know the worst case scenario when the structure is optimized for the nominal load directions and must minimize the consequences of directional uncertainties in the structure optimization process. Easy to imagine that, in extreme cases, even a slight directional perturbation of the nominal loads may cause a structure to collapse. Therefore, a conception, which tries to manage this problem, may be an efficient and viable tool in the structure optimization. Theoretically an engineering optimization problem can be characterized by an objective function (a performance measure) defined in the design space and a constraint set defined in the response space. A design is feasible when its response satisfies all constraints in the response space. Generally there is a nonlinear relation between the two spaces and a nonlinear algorithm is needed to get the “best” design. In the traditional deterministic case, there is a one-to-one relationship between the two spaces because the applied loads are represented by fixed vectors in the design space. In the case of the directionally uncertain loads, the feasible perturbations of each nominal load direction have to be defined by additional linear or nonlinear relations. Fig. 1 shows a directionally uncertain 3D unit load with maximum 30° allowable declination in any directions which can be described by an appropriate subset of spherical coordinates.

It is very important to mention that the feasibility have to be completely redefined in the case of directionally uncertain loads. In this case a design is only feasible when its feasibility is invariant to the directional perturbations, which theoretically can be proved by the evaluation of uncountable infinite number of additional constraints. It will be shown that this problem may be resolved by introducing a worst-load-direction-oriented approach, where worst-load-direction means a feasible direction, in which the maximum of the constraint violation measures is maximal, where the relative percentage error may be an appropriate constraint violation measure. By definition, when a design does not violate a constraint with an allowable tolerance than its measure value will be zero. The essence of the proposed approach is very simple: starting from the optimal nominal solution and the deterministic constraint set, in a cyclically repeatable process the algorithm identifies the currently worst direction; defines an additional constraint set for the corresponding load and solve the extended (multi-load) optimization problem; the process terminates after a finite number of steps when in the currently worst direction the maximum of the constraint violation measures will be zero. It is important to note that in the presented feasibility-oriented approach the objective function is not affected by the varying load direction; it remains the