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Research paper

Two-dimensional fracture modeling with the generalized/extended finite element method: An object-oriented programming approach

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ABSTRACT

This work presents an object-oriented implementation of the G/XFEM to model the crack nucleation and propagation in structures made of either linear or nonlinear materials. A discontinuous function along with the asymptotic crack-tip displacement fields are used to represent the crack without explicitly meshing its surfaces. Different approach are explained in detail that are used to capture the crack nucleation within the model and also determine the crack propagation path for various problems. Stress intensity factor and singularity of the localization tensor (which provides the classical strain localization condition) can be used to determine the crack propagation direction for linear elastic materials and nonlinear material models, respectively. For nonlinear material model, the cohesive forces acting on the crack plane are simulated in the enrichment process by incorporating a discrete constitutive model. Several algorithms and strategies have been implemented, within an object-oriented framework in Java, called INSANE. This implementation will be presented in detail by solving different two-dimensional problems, for both linear and nonlinear material models, in order to show the robustness and accuracy of the proposed method. The numerical results are compared with the reference solutions from the analytical, numerical or experimental results, where applicable.

1. Introduction

Fracture analysis using standard finite element method (FEM) is quite limited. In order to model crack propagation, remeshing is always needed to match the new geometry of the crack, except for some techniques such as smeared cracking approaches [22] that do not require remeshing. Generalized or extended FEM (G/XFEM) has been proposed to facilitate the modeling of arbitrary crack geometry and its evolution. It also eliminates need for remeshing and conformity to element boundaries. In G/XFEM [16,80], as in the FEM, the approximation is built over a mesh of elements using interpolation functions. Special functions multiply the original FEM functions and smooth as well as non-smooth solutions can be modeled independently of the mesh. In addition, there are other methods that can handle the crack propagation with their special techniques, such as: Mesh-free method [6,13], cracking particle method [5,75] or efficient remeshing techniques [4,48].

Application of object-oriented programming for FEM has been receiving great attention over the last two decades [56], for example: a FE analysis to solve structural problems using OOP approach within: Object NAP code [39], FEMOBJ [93], and OOFEM code [70]; a finite element differential equations analysis library [11]; an object-oriented environment to solve multidisciplinary problems (combination of thermal, fluid dynamics, and structural different fields) [27]; implementation of a unified library of nonlinear solution schemes in FE programming scheme [54]. Beside this, the OOP has been successfully used to represent also different numerical methods, such as the boundary element method [51] and meshfree methods [12]. Also, a bunch of G/XFEM codes used object-oriented concept as their implementation strategy: an extension of a FEM code by adding the G/XFEM enrichment strategy [81,82]; demonstration of an open source architecture for G/XFEM so-called openxfem++ [19]; an extension the OOFEM code to include the G/XFEM method [24]; implementing a G/XFEM code from scratch [35]; automated meshing for integrated experiments project proposed by Dunant [34]; and a G/XFEM implementation in *Python* by Neto et al. [68].

Initial implementation of the G/XFEM for crack propagation problems was introduced by Belytschko and Black [16] and Moës et al. [67]. After that, this method has been extensively used for the simulation of crack propagation problems, such as: three-dimensional crack propagation was proposed by Duarte et al. [33]; a quasi-static crack growth was proposed by Sukumar and Prévost [83] using G/XFEM;

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three-dimensional modeling of initiation, branching, growth of crack in non-linear solids including statics and dynamics problems was presented by Bordas et al. [20]; Rabczuk et al. [76] proposed some crack tracking techniques for three-dimensional problems using partition of unity and meshfree technique; and an enhanced G/XFEM method for modeling of dynamic crack branching presented by Xu et al. [90]. Dolbow et al. [30] modeled the fracture in Reissner-Mindlin plate with XFEM and computed the mixed-mode stress intensity factor using the domain forms of interaction integral. Subsequently, different researchers have used G/XFEM formulations based on Reissner-Mindlin plate theory to develop fracture modeling in shell and plate structures, see for example [14,52,65,92].

Although, there are many academic code implementing the G/XFEM approach for either static, quasi-static or dynamic analyses, as discussed earlier, but few of them address detail implementation aspects, such as openxfem+ + [19] (for stationary and crack propagation problems), OOFEM code [24] (for stationary crack problems), and G/XFEM in [3] (for stationary crack problems). However, none of them considered the crack nucleation into the problems. The work presented in [19] was almost an extension library which was designed to add XFEM capabilities to an existing FEM code, FEMOBJ [93]. The challenging issue is the integration of new implementation with an existing code. Chamrová and Patzák [24] presented an object-oriented approach implementation of the G/XFEM by extending the OOFEM, an available FEM code. The object-oriented structure of the this code was described in detail, including the role of individual classes and their mutual relations. In addition, an available FEM programming environment was expanded to enclose the standard version of the G/XFEM in [3], to analyze the static problems only for linear materials. This environment, so-called INSANE (INteractive Structural ANalysis Environment)[1] is an open source software available at http://www.insane.dees.ufmg.br.

The aim of this work is to fill this lack in the literature by presenting a new computational framework for crack nucleation and propagation that covers both linear and nonlinear material models. The whole implementation is based on object-oriented programming with capabilities of the G/XFEM that facilitates the expansion of the current implementations for other numerical approaches, such as meshless method. The advantages of the current OOP design as well as the whole implementations are discussed in detail with the use of various OOP features, such as abstract classes and single/multiple inheritances. A discontinuous function along with the asymptotic crack-tip displacement fields are used to represent the crack without explicitly meshing its surfaces. Different approach are explained in detail that are used to capture the crack nucleation within the model and also determine the crack propagation path for various problems. Stress intensity factor and singularity of the localization tensor (which provides the classical strain localization condition) can be used to determine the crack propagation direction for linear elastic materials and nonlinear material models, respectively. For nonlinear material model, the cohesive forces acting on the crack plane are simulated in the enrichment process by incorporating a discrete constitutive model. This constitutive model is defined as the traction-displacement relation in the crack path and is based on the concept of cohesive crack. An outline of the present paper is as follows. A general formulation of the classical G/XFEM is presented in Section 2. Section 4 presents a crack nucleation criterion and related formulations for fracture modeling of nonlinear medium. Section 5 provides explanation along with corresponding formulation for crack propagation process. The object-oriented implementation environment, INSANE, and its explanation for the present work is discussed in Section 6. In Section 7, the fracture modeling approach is applied to different problems which emphasizes the main ideas of the current implementations, and concluding remarks are brought in the final section.

2. Generalized/extended FEM

The G/XFEM was developed for modeling structural problems with discontinuities [16,32,33,64]. Furthermore, it can be considered an instance of the Partition of Unity Method, PUM [10], in the sense that it employs a set of Partition of Unity, PU, functions to guarantee inter-element continuity. Such strategy creates conforming approximations which are improved by a nodal enrichment scheme. The enrichment scheme is obtained by multiplying a PU function of C^0 type with compact support ω_j by the function $L_{ji}(\boldsymbol{x})$, named as a local approximation (also called enrichment function). The resulting shape function $\phi_{ji}(\boldsymbol{x})$ inherits characteristics of both functions, i.e., the compact support and continuity of the PU and the approximate character of the local function.

As a consequence, the generalized global approximation, denoted by $\tilde{\boldsymbol{u}}(\boldsymbol{x})$, can be described as a linear combination of the shape functions associated with each node:

$$\tilde{\boldsymbol{u}}(\boldsymbol{x}) = \sum_{j=1}^{N} \mathcal{N}_j(\boldsymbol{x})\left\{\boldsymbol{u}_j + \sum_{i=2}^{q} L_{ji}(\boldsymbol{x})\boldsymbol{b}_{ji}\right\} \tag{1}$$

where $\boldsymbol{u}_j$ is a nodal parameter associated with standard FE shape function- $\mathcal{N}_j(\boldsymbol{x})$, $\boldsymbol{b}_{ji}$ is nodal parameter associated with G/XFEM shape functions- $\mathcal{N}_j(\boldsymbol{x})\cdot L_{ji}(\boldsymbol{x})$. The enrichment function can be either continuous or discontinuous function, depending on the problem type. An example of the enrichment function, L_{ji}, by considering the singularities can be defined as [32]:

$$^{x}L^{s}_{j\alpha}(\boldsymbol{x})|_{\alpha=1} = \frac{1}{2G} r^{\lambda_1}\{[\kappa - Q_1(\lambda_1+1)]\cos\lambda_1\theta - \lambda_1\cos(\lambda_1-2)\theta\} \tag{2}$$

$$^{y}L^{s}_{j\alpha}(\boldsymbol{x})|_{\alpha=1} = \frac{1}{2G} r^{\lambda_1}\{[\kappa + Q_1(\lambda_1+1)]\sin\lambda_1\theta + \lambda_1\sin(\lambda_1-2)\theta\} \tag{3}$$

where r and θ are the polar coordinates centered on the crack tip, $\lambda_1 = 0.5, Q_1 = 1/3$ are coefficients of the first term of the solution in the neighborhood of the crack-tip, considering only the mode-I displacement field. Also, $\kappa = 3 - 4\nu$ and $\kappa = \frac{3-\nu}{1+\nu}$ for plane strain and plane stress analysis, respectively, and $G = \frac{E}{2(1+\nu)}$. The superscripts x and y are referred to $x-$ and $y-$directions, respectively. Another example of the enrichment function in which called the near-tip enrichment, is defined as [16]:

$$[F_i(r,\theta)] = \left[\sqrt{r}\sin\frac{\theta}{2},\ \sqrt{r}\cos\frac{\theta}{2},\ \sqrt{r}\sin\frac{\theta}{2}\sin\theta,\ \sqrt{r}\cos\frac{\theta}{2}\sin\theta\right] \tag{4}$$

where i is the number of crack-tip functions $F(r, \theta)$ and (r, θ) denotes the local polar coordinate defined at the crack-tip.

3. Reissner–Mindlin plate formulation

A structural element which is thin and flat is called *plate*. The *thin* means that the plate transverse dimension, or thickness, is small compared to the length and width dimensions. The Reissner–Mindlin plate theory are applied to very thin, $L/t > 100$, moderately thin, $20 < L/t < 100$, plates, where t and L represent the plate thickness and a representative length or width dimension. Reissner–Mindlin plate theory assumes that the normals to the plate do not remain orthogonal to the mid-plane after deformation, thus allowing for transverse shear deformation effects. This allows to use C^0 continuous elements.

The assumptions of the Reissner–Mindlin plate theory are the following: (1) the points belonging to the middle plane ($z = 0$), $u = v = 0$, which means the points on the middle plane, only move vertically; (2) the points along a normal to the middle plane have the same vertical displacement (i.e., the thickness does not change during deformation);

[1] The source code is available at the Git repository at https://git.insane.dees.ufmg.br/insane/insane.git.