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Brief paper

# Fault-tolerant control of Markovian jump stochastic systems via the augmented sliding mode observer approach☆

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ABSTRACT

This paper is concerned with the stabilization problem for a class of Markovian stochastic jump systems against sensor fault, actuator fault and input disturbances simultaneously. In the proposed approach, the original plant is first augmented into a new descriptor system, where the state vector, disturbance vector and fault vector are assembled into the state vector of the new system. Then, a novel augmented sliding mode observer is presented for the augmented system and is utilized to eliminate the effects of sensor faults and disturbances. An observer-based mode-dependent control scheme is developed to stabilize the resulting overall closed-loop jump system. A practical example is given to illustrate the effectiveness of the proposed design methodology.



## 1. Introduction

In modern industrial systems, various types of malfunction or imperfect behavior, resulted from the unexpected variations in normal wear in components, external surroundings, or sudden changes in signals, always occur inevitably in normal operations. These phenomenon are often referred as sensor/actuator faults. Since the fault can deteriorate system performances and even cause catastrophic accidents, it is of great importance to detect faults in time for the safety and reliability of control systems. In order to improve efficiency, the design strategies can be classified into fault detection and isolation (FDI) and fault-tolerant control (FTC). Over the past two decades, FDI and FTC have been extensively investigated, and a large number of results have been reported on FDI and FTC for various types of systems, see, for instance Basin and Rodriguez-Ramirez (2011), Jiang, Staroswiecki, and Cocquempot (2006), Niu and Ho (2010), Niu, Wang, and Wang (2010), Qi, Zhu, and Jiang (2013), Yin, Ding, Haghani, Hao, and Zhang (2012), Yin, Luo, and Ding (2013) and the references therein. To mention a few, the authors in Yin et al. (2013) were the first to propose a model-data integrated fault tolerant control scheme with performance optimization for real time industrial applications.

Markovian jump systems (MJSs) including both time-evolving and event-driven mechanisms have been applied to model the abrupt phenomena such as random failures, repairs of the components and sudden environment changes. A great number of efforts have been made to investigate the issues of stability, stabilization, and filtering of MJSs (Liu, Gu, & Hu, 2011; Shi, Boukas, & Agarwal, 1999a,b; Shi, Xia, Liu, & Rees, 2006; Shu, Lam, & Xiong, 2010; Wu & Ho, 2010; Wu, Yao, & Zheng, 2012; Zhang & Boukas, 2009). Another research frontier, stochastic differential equation (Basin, Elvira-Ceja, & Sanchez, 2011; Basin, Loukianov, & Hernandez-Gonzalez, 2010) has been also recognized as one of the most effective stochastic models in applications, e.g., aircraft, chemical or process control system, and distributed networks. Therefore, extensive attention has also been devoted to stochastic systems governed by Itô stochastic differential equations due to their numerous applications in mechanical systems, economics

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and systems with human operators. Furthermore, a variety of results have been reported with respect to the problems of stability, stabilization and filtering for Itô stochastic systems with or without Markovian jump (Niu, Ho, & Li, 2010; Niu et al., 2010; Shen, Wang, Shu, & Wei, 2008; Tong, Li, Li, & Liu, 2011; Wang, Ho, Dong, & Gao, 2010; Wang, Liu, & Liu, 2010).

It should be pointed out that, in practical engineering, Brownian motion, output disturbances, sensor and actuator faults often occur in control systems simultaneously. It is no doubt that the existences of these unexpected phenomena will deteriorate the performances or lead to instability of the systems. Therefore, the control systems are required to own the abilities to eliminate those effects and guarantee the stability of the corresponding closed-loop system. Although some attempts on FTC for Itô stochastic systems involved with Brownian motion have been made, the achieved results in the existing literature have been either focused only on sensor faults (Wang, Chien, & Lee, 2011; Wang, Chien, Leu, & Lee, 2010), or only on actuator faults (Jiang, Zhang, & Shi, 2011; Zhang, Jiang, & Shi, 2012; Zhang, Jiang, & Staroswiecki, 2010). However, in realistic industrial process, sensor fault and actuator faults always exhibit simultaneously, and more novel and effective control methodologies are desirable to be developed to solve the corresponding stabilization problems. This observation motivate us to address the fault diagnosis/estimation problem for Itô stochastic systems with Markovian switching under the considerations of combined faults and disturbances.

In this paper, the FTC problem is studied for a class of stochastic systems with Markovian jump parameters. The issues involved here are sensor and actuator faults, and output disturbances. A novel sliding mode observer approach is developed to solve the FTC problem for stochastic systems with Markovian jump parameters. By an augmented approach, the sensor faults and output disturbances are converted into "input disturbances' in the new augmented system. A sliding mode observer method is proposed to eliminate the effects of sensor and actuator faults, output disturbances simultaneously. Based on the estimation, a FTC strategy is synthesized to stabilize the resulting control system. Finally, a practical example is given to demonstrate the effectiveness of the proposed results. The remainder of the paper is organized as follows: the problem to be addressed is formulated in Section 2, and the main results are presented in Section 3. A design example is provided in Section 4 to demonstrate the effectiveness of the developed approach. Finally, Section 5 concludes the paper.

*Notations*: Throughout the paper, $\|\cdot\|$ and $|\cdot|$ denote the Euclidean norm and 1-norm of a vector, respectively. Given a symmetric matrix $A$, the notation $A > 0(< 0)$ denotes A is a positive definite matrix (negative definite, respectively). Given a square matrix $\tilde{A}$, $\tilde{A}^{\dagger}$ denotes the generalized inverse of $\tilde{A}$. $I_n$ denotes an identity matrix with dimension $n$. $\mathbb{R}^{+}$ denotes the set of all positive real numbers. $\mathcal{C}_{+}$ denotes the set of all complex numbers with positive real part.

## 2. Problem formulation

Let $\{r_t,\ t \geq 0\}$ be a homogeneous finite-state Markovian process with right continuous trajectories, which takes value in a finite state space $\mathcal{S} = \{1, 2, \ldots, s\}$ with generator $\Pi = [\pi_{ij}]$, $i, j \in \mathcal{S}$ given by

$$\Pr\left(r_{t+\triangle t} = j \mid r_t = i\right) = \begin{cases} \pi_{ij}\triangle t + o(\triangle t), & i \neq j \\ 1 + \pi_{ii}\triangle t + o(\triangle t), & i = j \end{cases}$$

where $\triangle t > 0$, and $\lim_{\triangle t \to 0}\left(\frac{o(\triangle t)}{\triangle t}\right) = 0$. $\pi_{ij} > 0$ for $i \neq j$ denotes the transition rate from mode $i$ to mode $j$ with $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$ for $i \in \mathcal{S}$. Consider the following Itô stochastic systems with Markovian jump parameters:

$$\begin{cases} \mathrm{d}x(t) = [A(r_t)x(t) + B(r_t)u(t) + B_a(r_t)f_a(t)]\,\mathrm{d}t \\ \qquad\quad + B_w(r_t)x(t)\,\mathrm{d}\omega(t), \\ y(t) = C(r_t)x(t) + D(r_t)u(t) + D_a(r_t)f_a(t) \\ \qquad\quad + D_d(r_t)d(t) + C_s(r_t)f_s(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state variable, $u(t) \in \mathbb{R}^m$ denotes the control input, and $y(t) \in \mathbb{R}^p$ is the measurement output. In system (1), $A(r_t) \in \mathbb{R}^{n\times n}$, $B(r_t) \in \mathbb{R}^{n\times m}$, $B_a(r_t) \in \mathbb{R}^{n\times a}$, $B_w(r_t) \in \mathbb{R}^{n\times n}$, $C(r_t) \in \mathbb{R}^{p\times n}$, $D(r_t) \in \mathbb{R}^{p\times m}$, $D_a(r_t) \in \mathbb{R}^{p\times a}$, $D_d(r_t) \in \mathbb{R}^{p\times d}$ and $C_s(r_t) \in \mathbb{R}^{p\times q}$ are system, $f_a(t) \in \mathbb{R}^a$ and $f_s(t) \in \mathbb{R}^q$ are the unknown actuator and sensor faults respectively, and $d(t) \in \mathbb{R}^d$ is the bounded disturbance. For notational simplicity, when $r_t = i$, $i \in \mathcal{S}$, the matrices $A(r_t)$, $B(r_t)$, $B_a(r_t)$, $B_w(r_t)$, $C(r_t)$, $D(r_t)$, $D_a(r_t)$, $D_d(r_t)$ and $C_s(r_t)$ will be represented by $A_i$, $B_i$, $B_{ai}$, $B_{wi}$, $C_i$, $D_i$, $D_{ai}$, $D_{di}$ and $C_{si}$ respectively. $\omega(t)$ denotes a standard one-dimensional Brownian motion on a probability space $(\Omega, \mathcal{F}, \mathcal{P})$ relative to an increasing family $(\mathcal{F}_t)_{t>0}$ of $\sigma$-algebra $\mathcal{F}_t \in \mathcal{F}$, in which $\Omega$ is the sample space, $\mathcal{F}$ is the $\sigma$-algebra of subsets of the sample space, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\omega(t)$ satisfies $\mathcal{E}\{\omega(t)\} = 0$, and $\mathcal{E}\left\{\omega^2(t)\right\} = t$.

The following assumptions are used throughout the paper.

(A1) The actuator vector $f_a(t)$, the sensor fault vector $f_s(t)$ and the disturbance $d(t)$ satisfy:

$$\|f_s(t)\| \leqslant \gamma_1, \quad \|\dot{f}_s(t)\| \leqslant \gamma_2, \quad \|f_a(t)\| \leqslant \alpha_1, \quad \|\dot{f}_a(t)\| \leqslant \alpha_2, \quad \|d(t)\| \leqslant d_1, \tag{2}$$

where $\gamma_1 > 0, \gamma_2 > 0, \alpha_1 > 0, \alpha_2 > 0$ and $d_1 > 0$ are known constants.

(A2) For each $i \in \mathcal{S}$, $(A_i,\ C_i)$ is an observable pair. In addition, there exists a scalar $\theta_i > 0$ such that the following rank condition holds

$$\operatorname{rank}\left[\begin{array}{c|c} \theta_i I_n + A_i & B_{ai} \\ \hline C_i & D_{ai} \end{array}\right] = n + a.$$

(A3) For each $i \in \mathcal{S}$, $D_{ai},\ D_{di}$ and $C_{si}$ are full column rank matrices.

**Remark 1.** The aforementioned three assumptions are reasonable and nonconservative. In practice, the real functions of sensor fault and actuator fault are often unknown. In order to present the FTC procedure, we provide assumption (A1), in which the condition is general in the existing FTC results. In addition, the condition in assumption (A2) is used for designing the observer and the condition in assumption (A3) is utilized for developing the sliding mode controller.

In the following discussion, we will present a new control approach for system (1) to obtain the estimations of $x(t)$, $f_s(t)$ and $f_a(t)$ simultaneously. In addition, an observer-based FTC scheme is developed to stabilize the closed-loop system.

The following preliminaries are introduced, which will be used for deriving our main results in the sequel.

**Definition 1** (*Mao, 1999*)**.** The stochastic system (1) with $u(t) \equiv 0$ is said to be stochastically stable if for every initial condition $x_0 \in \mathbb{R}^n$ and initial mode $r_0$, $\mathcal{E}\left\{\int_0^{\infty}\|x(t)\|^2\,\mathrm{d}(t) \mid x_0\right\} < \infty$ holds.

**Lemma 1** (*Chen, 1999*)**.** *Given a pair of matrix $(\tilde{A}, \tilde{C})$ with $\tilde{A} \in \mathbb{R}^{n\times n}$, $\tilde{C} \in \mathbb{R}^{p\times n}$, the following two conditions are equivalent: (i) The matrix $\tilde{A}$ is stable; (ii) If the pair $(\tilde{A},\ \tilde{C})$ is observable, then the Lyapunov equation $\tilde{A}^T\tilde{P} + \tilde{P}\tilde{A} = -\tilde{C}^T\tilde{C}$ has a unique solution.*