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Stabilization of sampled-data nonlinear systems by receding horizon control via discrete-time approximations☆

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Abstract

Results on stabilizing receding horizon control of sampled-data nonlinear systems via their approximate discrete-time models are presented. The proposed receding horizon control is based on the solution of Bolza-type optimal control problems for the parametrized family of approximate discrete-time models. This paper investigates both situations when the sampling period T is fixed and the integration parameter h used in obtaining approximate model can be chosen arbitrarily small, and when these two parameters coincide but they can be adjusted arbitrary. Sufficient conditions are established which guarantee that the controller that renders the origin to be asymptotically stable for the approximate model also stabilizes the exact discrete-time model for sufficiently small integration and/or sampling parameters.
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1. Introduction

The stabilization problem of nonlinear systems has received considerable attention in the last decades. The use of computers in the implementation of the controllers necessitated the investigation of sampled-data systems. One way to design a sampled-data control is to implement a continuous-time algorithm with sufficiently small sampling intervals. This approach is proposed in connection with the receding horizon method among the others in Fontes (2001), Jadababaie and Hauser (2001), Chen, Ballance, and O'Reilly (2000). However, some difficulty may arise during the application of this method: (1) the exact solution of the nonlinear continuous-time model is typically unknown, therefore an approximation procedure is unavoidable; (2) it may be difficult to implement an arbitrarily time-varying control function. An overview and analysis of existing approaches for the stabilization of sampled data systems can be found in the recent papers (Nešić, Teel, & Kokotović, 1999a; Nešić & Teel, 2004b, see also the references therein). In these papers, a systematic investigation of the connection between the exact and approximate models are carried out and conditions are presented which guarantee that the same family of controllers that stabilizes the approximate discrete-time model also practically stabilizes the exact discrete-time model of the plant both for the cases of fixed sampling period and varying integration parameter (Nešić & Teel, 2004b) and for the case when these two parameters coincide (Nešić et al., 1999a; Nešić & Teel, 2004b). Controller design within this framework is also addressed in Nešić and Teel (2004b), Laila and Nešić (2003), Nešić and Teel (2004a) and Nešić and Loria (2004).

There are several ways to design controllers satisfying the conditions given in Nešić et al. (1999a), Nešić and Teel (2004b). In Grüne and Nešić (2003), optimization-based methods are studied; the design is carried out either via an

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infinite horizon optimization problem or via an optimization problem over a finite horizon with varying length. To relax the computational burden needed in the case of infinite horizon optimization and in the case of optimization over a varying time interval, the application of the receding horizon method offers good vistas. In Grüne and Nešić (2003), it was pointed out that the results presented in that paper were not directly applicable for receding horizon control. The receding horizon method obtains the feedback control by solving a finite horizon optimal control problem at each time instant using the current state of the plant as the initial state for the optimization and by applying “the first part” of the optimal control. The study of stabilizing property of such schemes has been the subject of intensive research in recent years. From the vast literature we mention here but a few (Mayne & Michalska, 1990; Chen & Allgöwer, 1998; Gyurkovics, 1996; De Nicolao, Magni, & Scattolini, 1998; Parisini, Sanguineti, & Zoppoli, 1998; Gyurkovics, 1998; Jadababaie & Hauser, 2001; Fontes, 2001; Hu & Linnemann, 2002; Gyurkovics, 2002) and we refer the reader to the excellent overview papers (Allgöwer, Badgwell, Qin, Rawlings, & Wright, 1999; De Nicolao, Magni, & Scattolini, 2000; Mayne, Rawling, Rao, & Scokaert, 2000) and to the references therein. A great majority of works deals either with continuous-time systems with or without taking into account any sampling or with discrete-time systems considering the model given directly in discrete-time. To the best of our knowledge, the only exceptions are the very recent works of Ito and Kunisch (2002), where the effect of the sampling and zero-order hold is considered assuming the existence of a global control Lyapunov function and of Magni and , where a sampled-data control is applied to the continuous time system without taking into account any approximation in the plant model. Relying partly on the results of Nešić et al. (1999a) and Nešić and Teel (2004b), the present work studies the conditions under which the stabilizing, receding horizon control computed for the approximate discrete-time model also stabilizes the exact discrete-time system both in the cases when the sampling period T is fixed, but the integration parameter h used in obtaining the approximate model can be chosen arbitrarily small and when these two parameters coincide, but they can be adjusted. It should be emphasized that these conditions concern directly the data of the problem and the design parameters of the method, but not the results of the design procedure. From a practical point of view, it is important to know whether the basin of attraction is sufficiently large when some stabilizing controller is applied. This set is frequently compared with that of the infinite horizon regulator. Similar to Nešić and Teel (2004b) and Grüne and Nešić (2003), we can only prove local result in the case of fixed sampling period. However, in the case of adjustable sampling parameter, we shall show that the basin of attraction contains any compact subset of the set of initial points which are practically asymptotically controllable to the origin with piecewise constant sampled controllers. (Thus, if the controllability property is semiglobal, then semiglobal practical asymptotic stability is achieved, as well.)

In what follows, the notation $\mathscr{B}_\Delta = \{z \in \mathbb{R}^p : \|z\| \leqslant \Delta\}$ will be used both in $\mathbb{R}^n$ and $\mathbb{R}^m$ and $\mathscr{K}$, $\mathscr{K}_\infty$ and $\mathscr{KL}$ denote the usual class-$\mathscr{K}$, class-$\mathscr{K}_\infty$ and class-$\mathscr{KL}$ functions (see, e.g., Nešić, Teel, & Sontag (1999b))

2. Preliminaries and problem statement

2.1. The models

Consider the nonlinear control system described by

$$\dot{x}(t) = f(x(t), u(t)), \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in U \subset \mathbb{R}^m$, $f : \mathbb{R}^n \times U \to \mathbb{R}^n$, with $f(0,0) = 0$, U is closed and $0 \in U$. Let $\Gamma \subset \mathbb{R}^n$ be a given compact set containing the origin consisting of all initial states to be taken into account. The system is to be controlled digitally using piecewise constant control functions $u(t) = u(kT) =: u_k$, if $t \in [kT, (k+1)T)$, $k \in \mathbb{N}$, where $T > 0$ is the sampling period.

Assumption A1. (i) The function f is continuous and for any pair of positive numbers (Δ', Δ'') there exist a $T^* > 0$ such that for any $x_0 \in \mathscr{B}_{\Delta'}$, $\bar{u} \in U \cap \mathscr{B}_{\Delta''}$ and $T \in (0, T^*]$ Eq. (1) with $u(t) \equiv \bar{u}$, and $x(0) = x_0$ has a unique solution on $[0, T]$ denoted by $\phi^{\mathrm{E}}(., x_0, \bar{u})$.

(ii) For any pair of positive numbers (Δ', Δ'') there exists an $L_f = L_f(\Delta', \Delta'')$ such that

$$\|f(x,u) - f(y,u)\| \leqslant L_f \|x - y\|,$$

for all $x, y \in \mathscr{B}_{\Delta'}$ and $u \in \mathscr{B}_{\Delta''}$.

Then, the exact discrete-time model of the system can be defined as

$$x_{k+1} = F_T^{\mathrm{E}}(x_k, u_k), \tag{2}$$

where $F_T^{\mathrm{E}}(x,u) := \phi^{\mathrm{E}}(T, x, u)$. (A discussion about the case of finite escapes can be found, e.g. in Nešić et al. (1999a).)

Remark 1. If Assumption A1 is valid, then F_T^{E} is continuous in x and u and it satisfies a local Lipschitz condition of the following type: for each $\Delta' > 0$ there exist $T^* > 0$ and $L_f > 0$ such that

$$\|F_T^{\mathrm{E}}(x,u) - F_T^{\mathrm{E}}(y,u)\| \leqslant e^{L_f T} \|x - y\|, \tag{3}$$

holds for all $u \in \mathscr{B}_{\Delta'}$, all $T \in (0, T^*]$, and all $x, y \in \mathscr{B}_{\Delta'}$.

We emphasize that F_T^{E} in (2) is not known in most cases, therefore the controller design can be carried out by means of an approximate discrete-time model

$$x_{k+1} = F_{T,h}^{\mathrm{A}}(x_k, u_k), \tag{4}$$

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