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Avoiding local minima in the potential field method using input-to-state stability

M. Guerra [a,*,1], D. Efimov [b,c], G. Zheng [b,c], W. Perruquetti [b,c]

[a] DCSD - ONERA Centre de Toulouse, 2 Avenue Edouard Belin, 31000 Toulouse, France
[b] Non-A team @ Inria, Parc Scientifique de la Haute Borne, 40 avenue Halley, 59650 Villeneuve d'Ascq, France
[c] CRIStAL UMR CNRS 9189, Ecole Centrale de Lille, 59650 Villeneuve d'Ascq, France

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ABSTRACT

Supported by a novel field definition and recent control theory results, a new method to avoid local minima is proposed. It is formally shown that the system has an attracting equilibrium at the target point, repelling equilibriums in the obstacle centers and saddle points on the borders. Those unstable equilibriums are avoided capitalizing on the established Input-to-State Stability (ISS) property of this multistable system. The proposed modification of the PF method is shown to be effective by simulation for a two variable integrator and then applied to a unicycle-like wheeled mobile robots which is subject to additive input disturbances.

1. Introduction

Path planning is the crucial problem to solve when dealing with navigation for mobile robotics. Very early results on the field (Lozano-Pérez & Wesley, 1979) date back to the beginning of second half of the last century but in the 1980s, when the amount of research increased, results were produced which are used up to the present day (Hwang & Ahuja, 1992a; Souissi et al., 2013).

Path planning can be interpreted in two different ways (Masehian & Sedighizadeh, 2007). First, motion planning, in which the path is computed *a priori* knowing the environment and the robot model to determine a collision-free path. In this case a solution can be evaluated for very complex scenarios but uncertainties (changing) in the models of the environment, or of the robot, could lead to failure. In this category, it is possible to find approaches based on Dijkstra or the A∗ algorithms (Dijkstra, 1959; Hart, Nilsson, & Raphael, 1968), Potential Field methods (Khatib, 1986) and Rapidly exploring Random Trees (RRT) (LaValle, 1998; LaValle & Kuffner, 2000). The second category is represented by the sensor based approaches (*reactive*), in order to avoid the *a priori* knowledge of the map and deal with unknown conditions; among them it is possible to list the Dynamic Window Approach (DWA) (Fox, Burgard, & Thrun, 1997), the Velocity Obstacles approach (Fiorini & Shillert, 1998), the Virtual Field Histogram (VFH) (Borenstein & Koren, 1991) and its modification VFH+ (Ulrich & Borenstein, 1998), the last two based on the Potential Field (PF) method (Rimon & Koditschek, 1992). It is straightforward to understand that a combination of the two categories is the best solution to the path planning problem for mobile robots and it is indeed the most adopted (Hwang & Ahuja, 1992b; Khatib, 1996; Rizano, Fontanelli, Palopoli, Pallottino, & Salaris, 2013; Siegwart & Nourbakhsh, 2004).

This work is inspired by a recent result (Angeli & Efimov, 2013), in which a notion of Input to State Stability (ISS) for systems evolving in Riemannian manifolds is presented. The method takes into account multiple disconnected invariant sets and it allows the robustness against external disturbances to be evaluated in this complex scenario.

Using this notion, a reactive obstacle avoidance technique for WMRs based on the PF method is hereby presented.

The main problem with the PF method is the appearance of local minima which block the WMR and prevent to achieve the task (Koren & Borenstein, 1991 and Section 2 for further details), thus the first contribution of this work is represented by a local minima avoidance technique along with an *ad hoc* defined field. Starting from the hypothesis of disjoint obstacles, common in the literature (Khatib, 1996), a twice differentiable PF is designed and the gradient of such a field is used as an input for a two variable integrator. As detailed later in the manuscript, under certain

* Corresponding author.
E-mail address: matteo.guerra@onera.fr (M. Guerra).
[1] The work has been performed while M. Guerra was at Ecole Centrale de Lille and part of the NON-A team.

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assumptions the system is shown to be Input-to-State-Stable (ISS) with respect to decomposable invariants sets (Angeli & Efimov, 2013). Formally proving the ISS property allows us to escape local minima and to guarantee the global attractiveness of the target point. The singularities are avoided adding a complementary input which plays on the fact that the appearance of any bounded perturbation does not compromise the ISS property (Khalil, 1992) and this result is formally proven in the paper.

The designed PF and local minima avoidance technique are applied to drive a unicycle like Wheeled Mobile Robot (WMR) subject to additive input disturbances to a target (*i.e.* the origin). The aim is to have the WMR to track the movement of the 2D particle. The stabilization and tracking problem for non-holonomic WMR has been previously treated in the literature (Guldner, Utkin, Hashimoto, & Harashima, 1995; Khatib, 1996; Panagou, Tanner, & Kyriakopoulos, 2011; Samson, 1993), often in obstacle free scenario. Here we present two approaches: the former one applies an output linearization technique (Oriolo, De Luca, & Vendittelli, 2002; Siciliano, Sciavicco, Villani, & Oriolo, 2009) and it is indeed the simplest. The controls obtained for the particle case is applied, with a simple change of coordinates, directly to the WMR control inputs; the inconvenient is that this approach does not allow us to control the robot orientation.

A second controller, and second contribution of this paper, is designed to control both linear velocity and orientation of the WMR. This controller assigns for the linear velocity the norm of the field gradient while the angular velocity command is regulated with a *finite-time* control similar to the one used in Guldner et al. (1995). It is formally shown that the finite time control robustly guarantees the convergence of the robot orientation to the gradient direction and simulations have been carried to illustrate the behavior.

The contribution of the paper can be summarized in *two* main points:

- A solution to the local minima appearance in the PF method based on Angeli and Efimov (2013).
- A *finite time* control able to robustly track the trajectories generated by the designed PF.

Moreover, an experimental part sees a Turtlebot 2 WMR avoiding obstacles in an office-like environment. Usually, obstacle avoidance methods (as the PF one) relied on ultrasonic sensor (Veelaert & Bogaerts, 1999) or infrared ones (Pathak & Agrawal, 2005) while the actual trend is to use camera devices or laser range finders; in this work we use a LIDAR device to localize the WMR in a map with unknown obstacles and to realize the avoidance.

The paper is organized as follows: Section 2 presents some related works, Section 3 recalls the results of Angeli and Efimov (2013), Section 4 contains the definition of the field, its properties along with the main result. Section 5 shows how to apply the result to a unicycle-like WMR, presents the finite time control, and shows simulations and experiments; the paper ends with a conclusive Section 6 in which the authors present also some future directions.

2. Related works

The Potential Field method was firstly introduced by Khatib (1985) and developed in its generalized modification in Krogh (1984). Originally designed for manipulators (other examples can be found in Hourtash & Tarokh, 2001 and Varsos & Luntz, 2006), it has been modified to drive a mobile robot along a potential field whose minimum is at the target and in which each obstacle generates an additional repellent force which drives the robot away from it. It has been shown that this solution, even though mathematically elegant and quite effective practically, has some drawbacks when special events occur (Koren & Borenstein, 1991). The main inconvenient with the method is the appearance of *local minima* which block the robot due to particular obstacle configurations. Rimon and Koditschek (1992) proposed a modification of the PF based on *navigation functions*: in an *n*-dimensional spherical space the adopted field had no other local minima than the target specified, supposing though the complete environment to be known a priori. Other solutions use a harmonic potential field proposed by Connolly, Burns, and Weiss (1990), Sato (1992), and the more recent (Masoud, 2009), in which the method computes solutions to Laplace's Equation in arbitrary *n*-dimensional domains to have local minima free field, and results in a weak form of Rimon and Koditschek (1992). In Volpe and Khosla (1990), a different field formulation and obstacle representation are considered: the potential field includes 2 superquadric functions, one for the obstacle avoidance and one for the approaching which result in an elliptic isopotential contour of the obstacles to model a large variety of shapes. Last flaw of the method is the possibility to miss the target in case of an obstacle too close to it. This problem called *Goals nonreachable with obstacles nearby* (GNRON), treated in Ge and Cui (2000), deals with the case in which the repulsive force generated by an obstacle close to the target generate a force higher than the attractive one, preventing the robot to accomplish the task. There are also methods which do not eliminate unwanted equilibriums but generate local forces, *Virtual Hill*, to escape the disturbing minimum as in Park and Cheol (2004).

Within the local planners directly derived from the PF approach, as mentioned above, the VFH method firstly presented in Borenstein and Koren (1991) (see also its more recent modifications Ulrich & Borenstein, 1998, 2000) represents also a widely used solution to real-time obstacle avoidance. The first experiments ran on WMRs showed the shortcomings inherited after the PF approach: presence of traps and local minima. Thus, hybrid modifications merging global and local planners, like VFH+, were proposed: starting from a grid map, evaluates the PF at each iteration for a subset of active cells of the map, builds an obstacle histogram and reduces it to a polar form to finally compute the velocity commands. Many other modifications (Chuang & Ahuja, 1998; Kim, Wang, Ye, & Shin, 2004; Ma, Zheng, Perruquetti, & Qiu, 2014; Nishimura, Tanaka, Wakasa, & Yamashita, 2011; Okamoto & Akella, 2013) have been proposed in order to overcome all the cited shortages. In Ma et al. (2014), a new smooth repulsive force for the field is proposed applying the *i-PID* control method but no proof is given about actions taken to avoid local minima. The paper by Chuang and Ahuja (1998) studies just some aspects of the potential field method as local planner focusing on the repulsive field and not addressing the local minima problem. The work of Kim et al. (2004) is the one more similar to the result of the authors and it is based on a potential field which is not smooth. In Okamoto and Akella (2013) the PF method is applied to drive a group of WMR to a goal; the design of the PF is made to accomplish this task avoiding local minima but very simple and standard control techniques are applied (PI control).

3. Preliminaries

For an *n*-dimensional C^2 connected and orientable Riemannian manifold M without boundary (it is assumed here that M can be embedded in a Euclidean space, thus $0 \in M$), let the map $f: M \times \mathbb{R}^m \to T_x M$ be of class C^1 ($T_x M$ is the tangent space), and consider a nonlinear system of the following form:

$$\dot{x}(t) = f(x(t), d(t)) \tag{1}$$

where the state $x \in M$ and $d(t) \in \mathbb{R}^m$. The input $d(\cdot)$ is a locally