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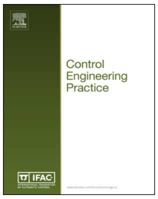

Design of experiments and statistical process control using wavelets analysis

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ABSTRACT

In this paper, three new connections between Wavelets analysis and Statistical Quality Control are proposed. Firstly, we show that the Discrete Wavelet Transform, using Haar wavelet, is equivalent to the Xbar-R control scheme. Results concerning the distribution of wavelets coefficients, using others wavelets families, are presented, and then a new control chart, called DeWave, is proposed, in order to monitor the variability of the process. Secondly, the equivalence between the Likelihood Ratio and the Continuous Wavelet Transform, in terms of estimating the change time, is presented. Finally, we demonstrate that the Discrete Wavelet Transform is an equivalent representation of factorial Design Of Experiments.

1. Introduction

Industrial systems are permanently monitored and controlled, in order to ensure product quality, reduce variability, and, in some cases, ensure safety operation (e.g. chemical process, nuclear power plant). Based on signals, which are generally collected from sensors, several techniques exist in order to detect abnormal events. Mathematically, this can be achieved by using statistical approaches and signal processing tools, such as wavelets analysis.

Statistical Quality Control (SQC) (Montgomery, 2005) gathers a set of methods for the control and optimization of industrial processes. Control charts are widely used to detect assignable causes in a process, and thus to reduce the production of nonconforming products or to avoid dangerous events. The Average Run Length (ARL) is generally used to evaluate the performance of these charts. The first control schemes (Xbar-R, Xbar-S) emerged in 1925, and proposed by Shewhart (1925). Afterwards, EWMA (Roberts, 1959) and CUSUM (Page, 1954) charts were proposed, which are more sensitive to small variations. Recently, ideas to vary control charts parameters (sample size, sampling interval, control limit coefficient) have been developed extensively by proposing several techniques, called adaptive control charts, for more details see Annadi, Keats, Runger, and Montgomery (1995), Reynolds, Amin, Arnold, and Nachlas (1988), Bai and Lee (1998), DeMagalhães, Epprecht, and Costa (2001), and Tiplica (2012). On the other hand, design of experiments is used to optimize the processes. It may be used either in process development of a new product or to obtain a process that is robust. Several types of design of experiments exist: factorial 2^k, fractional 2^{k-q}, and others (Montgomery, 2005).

Wavelets are a mathematical tool used to perform the analysis, the representation and the synthesis of signals. Wavelets transformation is a projection of the signal on the wavelets bases (Cohen, 1992; Daubechies, 1992; Mallat, 1999). Each dilation or compression of the Mother wavelet provides a scale decomposition. Each scale is composed of wavelets coefficients. They represent the signal at each scale (multi-scale decomposition). Several wavelets bases exist and are generally grouped by families (Haar, Daubechies, Symlet, etc.) (Daubechies, 1992). Today, areas of applications of wavelets are various: Signal and Image processing (Daubechies, 1990; JPEG2000), manufacturing (Gao & Yan, 2010), statistics (Abramovich, Bailey, & Sapatinas, 2000), and physics (Chanda, Kishore, & Sinha, 2005; Wharton, Wood, & Mellor, 2003). In process monitoring, wavelets are usually used as data preprocessing tool: *Thresholding* is a point of view widely used for handling data to improve FDI (Fault Detection and Isolation). In Jeong, Chen, and Lu (2003, 2006) authors discussed data reduction and noise reduction notions. They proposed a data reduction technique, which they compared with other techniques, such as Shrinkage. The Shrinkage is a set of de-noising techniques based on thresholding of wavelets coefficients (Donoho & Johnstone, 1994, 1995; Donoho, 1995). *Multi-scale analysis/Scalogram* can be

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Table 1
Hybrid (Statistical+Wavelets) process monitoring techniques.

Method	References
Principal component analysis+wavelets	Aradhye, Bakshi, Strauss, and Davis (2003), Bakshi (1998), Lee, Park, and Vanrolleghem (2005), Lu, Wang, and Gao (2003), Maulud, Wang, and Romagnoli (2006), Misra, Yue, Qin, and Ling (2002), Yoon and MacGregor (2004)
Kernel PCA+wavelets	Choi, Morris, and Lee (2008)
Independent component analysis+wavelets	Lin and Zhang (2005)
Neural networks+wavelets	Alexandridis and Zapranis (2013)

also used to characterize faults through scales and then classify them (Chanda et al., 2005; Liu et al., 2010). Many applications use the time–frequency representation, for example to identify the corrosion intensity the time–frequency plan was used to localize the corrosion (Dai, Motard, Joseph, & Silverman, 2000; Wharton et al., 2003). Wavelet-based methods for fault detection have been used for various applications, such as machine condition monitoring (Peng & Chu, 2004), bearing faults (Zarei & Poshtan, 2007), rotary machines (Yan, Gao, & Chen, 2014), gearbox failures (Fan & Zuo, 2006; Wang & McFadden, 1996), wind turbines (Sun, Zi, & He, 2014), compacts discs (Odgaard, Stoustrup, & Wickerhauser, 2006) and transmission lines (Liang, Elangovan, & Devotta, 1998). A very good literature review on Wavelet-based techniques for process monitoring is done here (Ganesan, Das, & Venkataraman, 2004).

Combination between wavelets and statistical techniques has been broadly developed in the multivariate context (see Table 1). Most of the published research papers that we found in the literature focus on the use of wavelets analysis as data pre-processing tool, such as de-nonising, data reduction, or feature extraction, in order to improve the detection performance.

The goal of this paper is to present some original equivalences between Wavelets transformations and statistical quality control techniques, such as control chart (fault detection), maximum likelihood estimator (fault isolation) and Design of Experiments (control optimization).

The paper is organized as follows: the second section notes the connection between the control charts (e.g. Xbar-R) and the Discrete Wavelet Transform (DWT); the third section presents the equivalence between the likelihood ratio test and the Continuous Wavelet Transform (CWT); and the fourth section concerns the equivalence between the factorial design of experiments 2^k and the DWT. Finally, the last section concerns conclusions and perspectives.

2. Wavelets & control charts

Discrete wavelets bases are defined as the discretization of scale s and translation T parameters of the continuous wavelets (Meyer, 1993; Daubechies, 1992; Cohen, 1992; Mallat, 1999). Multiresolution analysis (Mallat, 1989) is a framework that provides multi-scale decomposition across filter banks, which is defined by the scaling functions providing the approximation coefficients $a_j(k)$, and the wavelet functions providing the detail coefficients $d_j(k)$, they are defined as follows:

$$a_j(k) = \sum_{i=0}^{l} h[i]a_{j-1}[2k-i] \tag{1}$$

$$d_j(k) = \sum_{i=0}^{l} g[i]a_{j-1}[2k-i] \tag{2}$$

Where $a_0 = x$ is the original signal of size $N = 2^J$; $j \in \{1, 2, \ldots, J\}$; l: filter length, h: scaling function filter and g: wavelets function filter; $k \in \{1, 2, \ldots, 2^{J-j}\}$. The following theorem can be easily derived from the multi-resolution theory and the conditions to construct orthonormal compactly supported wavelets (Cohen, 1992; Daubechies, 1992; Gao & Yan, 2010). It concerns the probability distribution of wavelets coefficients (details and approximations). We show that wavelets coefficients present some interesting distributional characteristics that reflect the original data. This result will be exploited to propose new control charts.

Theorem 1. *Assume* $X = [x_1, x_2, \ldots, x_n]$ *a signal, where* $n = 2^J$, *and* x_i *are independent and identically distributed random variables, defined as follows:* $x_i \rightsquigarrow \mathcal{N}(\mu_0, \sigma_0^2)$. *Consider orthonormal and compactly supported wavelets (Haar, Daubechies, Symmlet, Coiflet, Discrete Meyer Wavelet). Multiresolution analysis of X provides wavelets coefficients distributed as follows:*

$$a_j(k) \rightsquigarrow \mathcal{N}(2^{j/2}\mu_0, \sigma_0^2)$$
$$d_j(k) \rightsquigarrow \mathcal{N}(0, \sigma_0^2)$$

which are independent and identically distributed random variables, $j \in \{1, 2, \ldots, J\}$.

Proof. The approximation wavelets coefficients, at the first scale, are

$$a_1(k) = \sum_{i=0}^{l} h[i]x[2k-i]$$

Then the expectation is defined as follows:

$$E(a_1) = E\left(\sum_{i=0}^{l} h[i]x[2k-i]\right) = \sum_{i=0}^{l} h_{i*}E(x) = \sqrt{2}_*\mu_0 = 2^{1/2}{}_*\mu_0$$

And the variance

$$V(a_1) = V\left(\sum_{i=0}^{l} h[i]x[2k-i]\right) = \sum_{i=0}^{l} h_i^2{}_*V(x) = V(x) = \sigma_0^2$$

By the same way, at the second scale

$$a_2(k) = \sum_{i=0}^{l} h[i]a_1[2k-i]$$

Then

$$E(a_2(k)) = E\left(\sum_{i=0}^{l} h[i]a_1[2k-i]\right) = \sum_{i=0}^{l} h_{i*}E(a_1(i)) = \sqrt{2}_*(\sqrt{2}_*\mu_0)$$
$$= 2^{2/2}{}_*\mu_0$$

And the variance

$$V(a_2(k)) = V\left(\sum_{i=0}^{l} h[i]a_1[2k-i]\right) = \sum_{i=0}^{l} h_i^2{}_*V(a_1) = V(a_1) = \sigma_0^2$$

By the same way, at the higher scales j, we conclude

$$E(a_j(k)) = 2^{j/2}{}_*\mu_0$$
$$V(a_j(k)) = \sigma_0^2$$

The detail wavelets coefficients are defined, at the first scale, as follows:

$$d_1(k) = \sum_{i=0}^{l} g[i]x[2k-i]$$