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Nonsmooth $\mathcal{H}_\infty$ synthesis of non-minimum-phase servo-systems with backlash

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ABSTRACT

The nonsmooth phenomena such as backlash and Coulomb friction, often occurring in mechanical systems, typically produce undesired inaccuracies, oscillations and instability thereby degrading the system performance. The present paper addresses these phenomena in a benchmark two-mass system composed of motor and load subsystems joined by an elastic shaft. Only partial measurements of the motor position are assumed to be available. The $\mathcal{H}_\infty$ synthesis is then developed for a class of nonsmooth systems with backlash. The effectiveness of the proposed synthesis and its robustness features in the presence of friction forces and backlash effects are supported by an experimental study made for an industrial emulator.

1. Introduction

Backlash is a common phenomenon in mechanical and hydraulic systems, which occurs due to the mechanical play between adjacent movable parts. The backlash phenomenon is accompanied with hysteresis between the input and output positions causing delays, oscillations, and impacts. Dry friction is another nonsmooth nonlinearity that appears commonly in mechanical systems. The backlash and dry friction effects on the system, such as inaccuracies, limit cycles, early wear of the mechanical parts degrading severely the system performance, and in the worst case could cause instability.

The backlash is often encountered in the input of the system due to imperfections between the actuator and the plant, and it is also typical for sandwich systems (two-mass systems), which include mechanical transmissions or gear trains being susceptible to imperfections. This paper considers backlash in a two-mass system, which results in an underactuated system constituted by two subsystems, the actuator and the load.

To deal with backlash problem, one can propose a mechanical solution, which consists of adding springs (pre-load-spring) to eliminate or reduce the effects of backlash; however, this method is not effective when rapid changes occur in the required direction (Lotfi, Zhong, & Khoo, 2010). Moreover, this is also an expensive solution, consumes energy, and increases the weight of the system (Grundelius & Angeli, 1996). Hence, solving this problem using a controller that involves the backlash behavior could be better than the mechanical solution.

Different models have been used to represent the backlash phenomenon. Some of these models include describing functions for the synthesis in the frequency domain as well as for systems with sinusoidal behavior (Freeman, 1959; Nakayama, Fujikawa, & Kobayashi, 2000). Another widely used model is the inverse model (Li, Su, & Chen, 2014; Tao & Kokotovic, 1993), which adds a block to compensate the effect of backlash when it appears at the input or output of the system. This model requires an accurate estimation of the parameters because an inadequate compensation (overcompensation or subcompensation) could lead to instability. Backlash can be also represented by a dynamical model as in Su, Stepanenko, Svoboda, and Leung (2000), requiring a precise knowledge of the parameters. The result is an approximation that omits an important part of the backlash behavior, and only numerical results are reported to support the use of this model (Guo, Wang, & Yao, 2012; Zhou, Wen, & Zhang, 2004). Static approximate models, proposed in Merzouki, Cadiou, and M'Sirdi (2004) and Zhao (2013), replace the nonsmooth nonlinearity by smooth ones thereby yielding persistent inaccuracies in the backlash representation. In Kolnik and Agranovich (2012), the backlash is viewed as a disturbance torque that requires an extra observer design for estimating the disturbance induced by the backlash. All these models represent backlash approximations, which ignore

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nonsmooth effects which are important in the feedback design to achieve accuracy in the closed-loop system behavior. In order to deal with an adequate representation of the transmitted torque with backlash, the deadzone model from Nordin, Galic', and Gutman (1997) and Lagerberg and Egardt (2003), which includes flexibility in the transmission or gear, is chosen for the present investigation.

A large variety of controllers used in systems with backlash can be found in the literature. We can find the well-known PI, PD or PID controllers (Dong & Mo, 2013; Mata-Jimenez & Brogliato, 2003; Mokhtari & Barati, 2006; Nakayama et al., 2000; Rodriguez-Linan & Heath, 2012), the widely used adaptive controllers when parameter uncertainties are in play (Guo et al., 2012; Liu et al., 2015; Tao & Kokotovic, 1993) as well as discontinuous (Corradini & Orlando, 2002; Merzouki et al., 2004; Orlov, Aguilar, & Cadiou, 2003), and fuzzy controllers (Liu & Tong, 2014; Wang, Liu, & Lai, 2015), among others (see, e.g., the survey paper Nordin & Gutman, 2002). Robust $\mathcal{H}_\infty$ synthesis of systems with backlash may also be found in Acho, Ikhouane, and Pujol (2013) where the standard linear $\mathcal{H}_\infty$ approach is utilized while treating backlash nonlinearities as bounded disturbances. The nonlinear $\mathcal{H}_\infty$ synthesis is applied in Aguilar, Orlov, Merzouki, and Cadiou (2007) to a servo-system regulation problem where the backlash phenomenon is modeled using an approximate model whereas (Ponce, Bentsman, Orlov, & Aguilar, 2013) solves the regulation problem for a specific system with backlash in the input using the dead-zone model.

In the existing literature on stabilization and regulation of systems with backlash, the dry friction phenomena are mostly ignored and only few results (Friedland, 1997; Lin, Yu, & Feng, 1996; Marton & Lantos, 2009) are available. Further investigation is thus motivated for tracking synthesis that would address backlash and dry friction phenomena in combination. Such a problem, becomes even more challenging if the full information on the system is no longer available that corresponds to a practical situation where no load measurements are normally feasible.

The objective of the present work is to extend the nonlinear $\mathcal{H}_\infty$ approach towards output feedback tracking of a nonsmooth servo-system, which is non-minimum phase due to the transmitted torque, passing from the motor to the load through a deadzone. The novelty of the conducted investigation is in managing robust synthesis under inaccuracies in modeling backlash phenomena while admitting only angular position measurement of the motor.

The load tracking problem of interest is solved by involving an appropriate switched reference trajectory of the motor that results in the desired load tracking. The nonsmooth $\mathcal{H}_\infty$ synthesis is then developed to asymptotically track the nominal motor motion while attenuating external disturbances and model uncertainties. Since the standard nonlinear $\mathcal{H}_\infty$ approach, as in Basar and Bernhard (1995), Isidori and Astolfi (1992) and van der Shaft (1992), does not admit a straightforward application under existing nonsmooth effects, the nonsmooth $\mathcal{H}_\infty$ synthesis, recently proposed in Orlov (2009), is invoked to deal with a nonsmooth representation of the system backlash and dry friction. To enhance the controller performance a dynamic filter of the switched motor reference trajectory is additionally involved into the closed-loop system and its influence on the system performance is experimentally studied in a laboratory testbed.

Along with the proposed $\mathcal{H}_\infty$ synthesis under the present circumstances, experimental verification of the good performance of the synthesized servo-system motion is the main contribution of the paper. Preliminary experimental results of this work were presented in Ponce, Orlov, Aguilar, and Alvarez (2015) and performance comparison of the developed nonsmooth $\mathcal{H}_\infty$ controller vs. a linear $\mathcal{H}_\infty$ controller was reported in Ponce, Orlov, Aguilar, and Alvarez (2015). The present paper refines the conference publications (Ponce et al., 2015a,b) by performing a deeper experimental study to validate the use of a motor reference filter and to verify the robustness features of the proposed synthesis against external disturbances and parametric uncertainties.

The rest of the paper is outlined as follows. In Section 2, the generic nonsmooth $\mathcal{H}_\infty$ synthesis is presented. Section 3 introduces a servo-system with backlash for which the load tracking problem is stated and resolved via motor position feedback $\mathcal{H}_\infty$ synthesis. Section 4 conducts an experimental study of an industrial emulator to support the theory. Finally, Section 5 collects some conclusions.

2. Preliminaries: generic nonsmooth $\mathcal{H}_\infty$ synthesis

For later use, the local $\mathcal{H}_\infty$ synthesis is recalled from Orlov (2009) for a generic nonautonomous nonsmooth system of the form

$$\begin{aligned}\dot{x}(t) &= f(x(t), t) + g_1(x(t), t)w(t) + g_2(x(t), t)u(t),\\ z(t) &= h_1(x(t), t) + k_{12}(x(t), t)u(t),\\ y(t) &= h_2(x(t), t) + k_{21}(x(t), t)w(t),\end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the disturbance attenuator, $w(t) \in \mathbb{R}^r$ is the unknown disturbance, $z(t) \in \mathbb{R}^l$ is the output to be controlled, $y(t) \in \mathbb{R}^p$ is the available measurement of the system.

The nonsmooth $\mathcal{H}_\infty$-control problem for a generic system (1) is to find a locally stabilizing output feedback controller

$$\begin{aligned}u &= \mathcal{K}(\xi, t)\\ \dot{\xi} &= \mathcal{F}(\xi, y, t),\end{aligned} \tag{2}$$

with internal state $\xi \in \mathbb{R}^s$ such that the $\mathcal{L}_2$-gain of the closed-loop system (1), driven by (2), is locally less than γ. Solving the above problem under γ approaching the infimal achievable level γ^* in (3) yields a (sub)optimal $\mathcal{H}_\infty$-controller with the (sub)optimal disturbance attenuation level γ^* ($\gamma > \gamma^*$).

For convenience of the reader recall that the generic system (1) locally possesses $\mathcal{L}_2$-gain less than γ iff the following inequality holds:

$$\int_0^T \|z(t)\|^2 \, dt < \gamma^2 \int_0^T \|w(t)\|^2 \, dt \tag{3}$$

for all $T > 0$, for all the system trajectories initialized in the origin, and for all piecewise continuous functions $w(t) \in \mathcal{L}_2(0, T)$ (particularly, constant disturbances are admitted) such that the state trajectories remain in a vicinity of the origin.

The following assumptions are imposed on the generic system (1):

A1: The functions $f(x, t)$, $g_1(x, t)$, $g_2(x, t)$, $h_1(x, t)$, $h_2(x, t)$, $k_{12}(x, t)$, $k_{21}(x, t)$ are continuous in t, and locally Lipschitz continuous in x for all t;

A2: For almost all $t \in \mathbb{R}$, there exists a neighborhood $U_t(0)$ of the origin $x=0$, possibly dependent on t, such that the functions, listed in Assumption A1, are uniformly bounded in t, twice continuously differentiable in x, and their first and second order state derivatives are piecewise continuous and uniformly bounded in $t \in \mathbb{R}$ for all $x \in U(0)$.

A3: $h_1^T(x, t)k_{12}(x, t) = 0$, $k_{12}^T(x, t)k_{12}(x, t) = I$, $k_{21}(x, t)g_1^T(x, t) = 0$, $k_{21}(x, t)k_{21}^T(x, t) = I$ for all t.

A4: $f(0, t) = 0$, $h_1(0, t) = 0$, and $h_2(0, t) = 0$ for all t.

Assumption A1 admits nonsmooth nonlinearities guaranteeing the well-posedness of the above dynamic system under integrable exogenous inputs. Assumption A2, made for a technical reason, allows one to locally linearize the closed-loop system, driven by a