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A systematic approach for robust repetitive controller design

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ABSTRACT

In this paper, a methodology for the synthesis of repetitive controllers to ensure periodic reference tracking and harmonic disturbance rejection is cast in a robust control framework. Specifically, the Lyapunov–Krasovskii theory is applied to derive LMI-based conditions for designing a state feedback control law with guaranteed stability and performance properties for system parameter variations. Practical experiments in commercial uninterruptible power supplies – UPS are considered to illustrate and discuss some practical implementation aspects of the proposed method.

1. Introduction

Reference tracking and disturbance rejection are two requirements that control engineers usually have to deal in many practical applications such as DC motors (Ramos, Cortes-Romero, & Coral-Enriquez, 2015; Wu, Xu, Cao, & She, 2014), anti-vibration systems (Yao, Tsai, & Yamamoto, 2013) and power inverters (del Puerto-Flores et al., 2014; Lopez Arevalo, Zanchetta, Wheeler, Trentin, & Empringham, 2010; Rohouma, Zanchetta, Wheeler, & Empringham, 2015), to cite a few. In particular, these references address the tracking and disturbance rejection problem of periodic signals, where traditional controllers such as Proportional-Integral-Derivative (PID) and Lead-Lag compensators cannot robustly guarantee tracking/rejection performance (Chen, 1970). Traditionally, in control system theory, the latter requirements are satisfied considering controllers based on the Internal Model Principle (IMP) (Chen, 1970), which basically establishes that the controller should contain the nonvanishing modes of reference and disturbance inputs, assuming closed loop stability.

The IMP is the basic principle considered by repetitive and resonant controllers to deal with periodic references and disturbances. Precisely, the perfect steady-state reference tracking or disturbance rejection of a sinusoidal signal with frequency ω_0 will be guaranteed only if the controller contain a pair of poles at $\pm\omega_0$ in the imaginary axis. The extension of the IMP to cope with periodic signals of any nature can be easily obtained by representing them in terms of a Fourier series expansion, i.e. to represent it as a weighted sum of sinusoidal terms whose frequencies are integer multiples (i.e. harmonics) of the fundamental frequency. Hence, the controller will contain several pairs of complex poles in the imaginary axis located at the fundamental frequency and its harmonics. Since the controller frequency response presents peaks with infinite gain at the resonance frequencies, these controllers are usually referred as resonant controllers (Angulo, Ruiz-Caballero, Lago, Heldwein, & Mussa, 2013). One of the main drawbacks of these controllers lies in its tuning complexity. Depending on the number of harmonic frequencies effectively considered, a large number of parameters have to be designed (Pereira, Flores, Bonan, Coutinho, & Gomes da Silva, 2014). An alternative and less complex control structure implementing the IMP for dealing with general periodic signals is the repetitive controller. The repetitive control structure is usually implemented by means of a delay element (related to its fundamental frequency) in a positive feedback loop leading to infinite resonant peaks in the controller frequency response (Chen & Tomizuka, 2014). Despite its effectiveness in solving the tracking and disturbance rejection problems, the delay element introduces some complexity on the control design of systems subject to parameter uncertainties.

A typical example of systems subject to periodic reference/

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disturbance signals and parameter uncertainties are uninterruptible power supplies (UPS) which are usually employed to protect critical loads against sudden disturbances or line failures. Due to the critical nature of loads, UPS have to satisfy severe quality requirements such as fast dynamic response, noise attenuation and low total harmonic distortion (THD) (IEC62040, 2004). The disturbance rejection problem is particularly interesting when UPS are feeding nonlinear loads with high harmonic content as, for instance, line rectifiers with large capacitive filters. Traditionally, PID controllers are the most common control technique employed in commercial UPS (Jaafar, Alawieh, Ortega, Godoy, & Lefranc, 2013; Willmann, Coutinho, Pereira, & Libano, 2007). In particular, a multi-loop approach is applied consisting of an inner PD loop to track a sinusoidal reference (fast dynamics) and an outer PI loop to adjust the root mean square (RMS) value of the output voltage. However, this classical PI–PD control structure generally yields high THD values when feeding nonlinear loads and the closed loop transient performance is affected by the RMS calculation which requires a full reference cycle to be performed (Willmann et al., 2007).

Repetitive controllers have been widely applied to control UPS systems because of its simple structure and small number of tuning parameters. For instance, a standard repetitive control structure is added to a negative feedback loop in Escobar, Valdez, Leyva-Ramos, and Mattavelli (2007) for compensating only odd harmonics of the fundamental frequency. A new structure combining repetitive and resonant controllers is proposed in Salton, Flores, Pereira, and Coutinho (2013) in order to reduce THD and improve the transient response of UPS systems. Digital implementations of repetitive controllers in the UPS context were developed in Buso and Mattavelli (2006), Rech, Pinheiro, Gründling, Hey, and Pinheiro (2003), and Lu, Zhou, Wang, and Cheng (2014). In Rech et al. (2003), a comparison between different algorithms to implement the repetitive controller is presented, while a novel repetitive structure optimizing data memory is presented in Lu et al. (2014).

This paper proposes a linear matrix inequality (LMI) based strategy to the synthesis of repetitive controllers for systems subject to periodic references/disturbances and parameter variations. More precisely, the preliminary results reported in Bonan, Flores, Coutinho, and Pereira (2011) are extended in several ways given special attention to application aspects. In particular, a detailed controller analysis is presented to demonstrate that the addition of a first order filter to the basic repetitive control structure impacts on the closed-loop transient and steady state responses. Then, a systematic procedure is proposed to determine the controller parameters with guaranteed robustness properties as well as transient performance to parameter variations. The controller parameters are obtained by means of a convex optimization problem which is numerically solved through available standard software packages such as Sturm (1999) and Toh, Todd, and Tutuncu (1999). Hence, simulation and experimental results are presented to illustrate the behavior of the proposed approach when applied to a commercial 4.3 kVA PWM single phase half bridge DC–AC inverter usually considered in the output stage of UPS. To critically evaluate the closed loop performance, linear and nonlinear loads are considered in the experimental setup according to the IEC 62040-3 standard.

This paper is organized as follows. In Section 2, the uncertain state-space model of the system under analysis is presented. Thus, a detailed explanation regarding the repetitive controller including the effects of the low-pass filter on the relative stability, steady-state error and dynamic response of the closed loop system is provided in Section 3. Based on Lyapunov–Krasovskii theory, LMI conditions are derived in Section 4 to synthesize the controller parameters such that robust closed-loop stability and performance are guaranteed. In Section 5, simulation and experimental results are presented to evaluate the behavior of the proposed solution, and some concluding remarks are drawn in Section 6.

Notation: $\mathbb{R}$ is the set of real numbers, $\mathbb{C}$ is the set of complex numbers, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of $n \times m$ real matrices, $\|\cdot\|$ is the Euclidean vector norm, $\mathbf{0}_n$ and $\mathbf{0}_{m\times n}$ are the $n\times n$ and $m \times n$ matrices of zeros, $\mathbf{I}_n$ is the $n\times n$ identity matrix. For a real matrix $\mathbf{S}$, $\mathbf{S}'$ denotes its transpose, and $\mathbf{S} > 0$ ($\mathbf{S} < 0$) means that $\mathbf{S}$ is symmetric and positive-definite (negative-definite). Matrix and vector dimensions are omitted whenever they can be inferred from the context. The time-derivative of a function $r(t)$ will be denoted by $\dot{r}(t)$ and the argument (t) is often omitted.

2. Preliminaries

Consider the following Single-Input Single-Output – SISO system described in state space by

$$\begin{cases} \dot{\mathbf{x}}(t) = \mathbf{A}(Y_0(t))\mathbf{x}(t) + \mathbf{B}u(t) + \mathbf{B}_d i_d(t) \\ y(t) = \mathbf{C}\mathbf{x}(t) \\ e(t) = r(t) - y(t) \end{cases} \tag{1}$$

where $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector, $y(t) \in \mathbb{R}$ is the output signal to be controlled, $r(t) \in \mathbb{R}$ is a periodic reference signal to be tracked by $y(t)$ and $i_d(t) \in \mathbb{R}$ is a periodic disturbance. Signals $r(t)$ and $i_d(t)$ are assumed to have the same fundamental period (here denoted by τ), however they can significantly differ with respect to harmonic content. Matrices $\mathbf{B}$, $\mathbf{B_d}$ and $\mathbf{C}$ are supposed to be constant with appropriate dimensions and $\mathbf{A}(Y_0(t))$ is a matrix function of the uncertain parameter $Y_0(t)$.

It is assumed that $Y_0(t)$ is bounded with known minimum and maximum values, that is:

$$\underline{Y}_0 \le Y_0(t) \le \overline{Y}_0.$$

For convenience, the parameter $Y_0(t)$ is often cast in terms of its nominal value $\tilde{Y}_0$ and deviation $\hat{Y}_0$ as follows:

$$Y_0(t) = \tilde{Y}_0 + \delta(t)\hat{Y}_0, \quad \delta(t) \in [-1, 1], \tag{2}$$

where

$$\tilde{Y}_0 = \frac{\underline{Y}_0 + \overline{Y}_0}{2} \quad \text{and} \quad \hat{Y}_0 = \frac{\overline{Y}_0 - \underline{Y}_0}{2}.$$

In view of (2), notice that the Linear Fractional Transformation (LFT) approach can be applied to describe $\mathbf{A}(Y_0(t))$ in the following form (Zhou & Doyle, 1998):

$$\mathbf{A}(Y_0(t)) = \mathbf{A}(\tilde{Y}_0) + \mathbf{H}(\hat{Y}_0)\delta(t)\mathbf{E}, \quad \delta(t) \in [-1, 1], \tag{3}$$

where $\mathbf{A}(\tilde{Y}_0)$, $\mathbf{H}(\hat{Y}_0)$ and $\mathbf{E}$ are constant matrices representing the uncertainty structure.

3. Repetitive controller

3.1. Basic concepts

The idea of repetitive control was initially proposed in Inoue, Nakano, and Iwai (1981) as an alternative to traditional controllers to ensure the reference tracking or disturbance rejection of periodic signals. Its working principle is to store the tracking error during a complete period and to feed the delayed error signal in the nominal system (Yamamoto, 1993). Typically, this behavior is obtained by the introduction of a delay element with value equal to τ in a positive feedback loop. This results in a transfer function