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Extended complex Kalman filter for sensorless control of an induction motor

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ABSTRACT

This paper deals with the design of an extended complex Kalman filter (ECKF) for estimating the state of an induction motor (IM) model, and for sensorless control of systems employing this type of motor as an actuator. A complex-valued model is adopted that simultaneously allows a simpler observability analysis of the system and a more effective state estimation. The observability analysis of this model is first performed by assuming that a third order ECKF has to be designed, by neglecting the mechanical equation of the IM model, which is a valid hypothesis when the motor is operated at constant rotor speed. It is shown that this analysis is more effective and easier than the one performed on the corresponding real-valued model, as it allows the observability conditions to be directly obtained in terms of stator current and rotor flux complex-valued vectors. Necessary observability conditions are also obtained along with the well-known sufficient ones. It is also shown that the complex-valued implementation allows a reduction of 35% in the computation time w.r.t. the standard real-valued one, which is obtained thanks to the lower dimensions of the matrices of the ECKF w.r.t. the ones of the real-valued implementation and the fact that no matrix inversion is required. The effectiveness of the proposed ECKF is shown by means of simulation in Matlab/Simulink environment and through experiments on a real low-power drive.

1. Introduction

Sensorless control of systems employing induction motors as actuators requires the estimation of the rotor speed together with the rotor flux components that cannot be directly measured. The estimation of the IM rotor speed can be performed by means of two types of methods, both exploiting the information from stator current measurement. The former type of method is based on the recognition of the characteristics (Holtz, 2002; Hurst & Habetler, 1996) of the measured currents, whereas the latter one is a model-based approach (Rajashekara, Kawamura, & Matsuse, 1996; Vas, 1998). The first type of methods can be invasive if based on the superimposition of suitable signals to the standard ones (Holtz, 2002), but not invasive if based on spectral analysis of the stator currents (Hurst & Habetler, 1996). They are also subject to interpretation errors of the above characteristics. The second type of methods use a priori knowledge of the actual system and are not invasive since they involve only measured variables. Moreover, their sensitivity to variations of the model's parameters or poor knowledge of their values can be countered by using robust estimation techniques. Among these model-based methods, MRAS-type (Cirrincione, Pucci, Cirrincione, & Capolino, 2004; Tajima & Hori, 1993), Luenberger-type (Cirrincione, Pucci, Cirrincione, & Capolino, 2006; Rajashekara et al., 1996), and sliding mode observers (Ghanes & Zheng, 2009; Rodic & Jezernik, 2002; Yan, Jin, & Utkin, 2000) are good examples of deterministic observers, whereas Kalman filters (Alonge, D'Ippolito, & Sferlazza, 2014) and extended Kalman filters (Alonge & D'Ippolito, 2010; Kim, Sul, & Park, 1994; Vas, 1998) are good examples of stochastic estimators.

In this paper the problem of designing a state observer for the IM consisting of an extended complex-valued Kalman filter is addressed, based on a complex-valued description of the dynamical behavior of the IM itself (cf. Menaa, Touhami, Ibtiouen, & Fadel, 2007; Petersen & Savkin, 1999; Vas, 1998). The model in question is of reduced order with state variables given by the complex stator current, the complex rotor flux, and a real variable representing the rotor speed. Related to the possibility of building such a state observer is the study of the observability property of the complex-valued model. In the standard real-valued framework this property has been largely studied since the seminal work of Canudas De Wit, Youssef, Barbot, Martin, and Malrait (2000) and later in Marino, Tomei, and Verrelli (2010), Ibarra-Rojas, Moreno, and Espinosa-Pérez (2004), and Ghanes, De Leon, and Glumineau

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(2006). When assuming a constant rotor speed, the adoption of the complex-valued model allows us to derive necessary observability conditions, directly obtained in terms of stator current and rotor flux complex variables, along with the well-known sufficient ones obtained for the real-valued IM model (Canudas De Wit et al., 2000; Marino et al., 2010).

Furthermore, it is shown that the sufficient conditions, ensuring the observability property of the continuous-time complex-valued IM model, are preserved by a first-order Euler discretization, which is essential to prove the feasibility of a discrete-time estimator. To this regard, the use of extended Kalman filters for state estimation of real-valued IM models, even in speed sensorless configuration, is a well recognized approach (see e.g. Alonge & D'Ippolito, 2010; Kim et al., 1994). Recently, techniques allowing the optimization of the parameters of such filters have been proposed, based on deterministic (Alonge & D'Ippolito, 2010) as well as stochastic approaches (Buyamin, 2007). From a complexity viewpoint, the proposed ECKF allows the implementation of a state observer with a substantial reduction (around 35%) of the required computation time. This reduction is achieved since all involved matrices have lower dimensions than those obtained from the corresponding real-valued models, and no matrix inversion is needed as the system output is a scalar variable represented by the complex-valued stator current. Moreover, the ability of our ECKF to produce accurate estimates of the IM state is shown by means of simulation in Matlab/Simulink environment. Its practical applicability is successfully tested by means of experiments on a real low power drive, where the output of the filter is given to a simple proportional–integral controller.

The paper is organized as follows. Section 2 describes the complex-valued model of the IM. Section 3 deals with observability analysis of the model and provides necessary and sufficient observability conditions for it. Section 4 shows that the sufficient observability conditions are preserved under time-discretization. Section 5 describes the ECKF. Section 6 shows simulation results of a sensorless closed loop control system in which the estimator consists of an ECKF. Finally, Section 7 presents the experimental results, which shows the effectiveness of the proposed ECKF.

2. Complex model of the induction motor

As is well known, the standard mathematical model of the IM in stationary frame is given by

$$\dot{i}_\alpha = -a_{11} i_\alpha + a_{12}\psi_\alpha + f_1\psi_\beta\omega + f_1 u_\alpha, \tag{1}$$

$$\dot{i}_\beta = -a_{11} i_\beta + a_{12}\psi_\beta - f_1\psi_\alpha\omega + f_1 u_\beta, \tag{2}$$

$$\dot{\psi}_\alpha = a_{21} i_\alpha - a_{22}\psi_\alpha - \psi_\beta\omega, \tag{3}$$

$$\dot{\psi}_\beta = a_{21} i_\beta - a_{22}\psi_\beta + \psi_\alpha\omega, \tag{4}$$

$$\dot{\omega} = -a_{33}\omega - f_3(i_\alpha\psi_\beta - i_\beta\psi_\alpha) - g_5 t_l, \tag{5}$$

where

$$a_{11} = \frac{1}{L_e}\left(R_s + \frac{L_s - L_e}{\tau_r}\right), \quad a_{12} = \frac{1}{\tau_r L_e}, \quad a_{21} = \frac{L_s - L_e}{\tau_r},$$
$$a_{22} = \frac{1}{\tau_r}, \quad a_{33} = \frac{F}{J_M}, \quad f_1 = \frac{1}{L_e}, \quad f_3 = \frac{2p^2}{3J_M}, \quad g_5 = \frac{p}{J_M},$$

and the other symbols are defined in Table 1.

Table 1
Parameters of the induction motor model.

i_α (i_β)	Stator current component along α-axis (β-axis) fixed to the stator, A
u_α (u_β)	Stator voltage component along α-axis (β-axis), V
ψ_α (ψ_β)	Scaled rotor flux along α-axis (β-axis), Wb
ω	Rotor speed, el. rad/s
R_s (L_s)	Stator resistance (inductance), Ω (H)
L_m (L_r)	Mutual (rotor) inductance, H
R_r	Rotor resistance, Ω
τ_r	$\left(=\frac{L_r}{R_r}\right)$ rotor time constant, s
L_e	$\left(=L_s - \frac{L_m^2}{L_r}\right)$ stator equivalent inductance, H
F	Viscous friction coefficient, N m s
t_l	Load torque, N m
J_M	Inertia coefficient, N m s^2
p	Pole pairs
T_s	Sampling time, s
$\|\cdot\|$	Euclidean vector norm, corresponding to the induced norm for matrices
I_n	Identity matrix of order n

Since our objective is that of designing an EKF that is able to estimate the whole system state starting from the measure of the stator currents, when speed varies slowly between two sample times, the mechanical equation can be neglected by putting the second member of (5) equal to zero. Moreover, with the aim of reducing the order of the model, the following complex state and input vectors are defined:

$$x = (x_1, x_2, x_3)^T = (i, \psi, \omega)^T = (i_\alpha + j i_\beta, \psi_\alpha + j\psi_\beta, \omega)^T,$$

and

$$u = u_\alpha + j u_\beta,$$

where j is the imaginary unit. In terms of the new complex state variables, choosing the complex current i as output and the complex stator voltage u as input, the dynamic model (1)–(5) becomes

$$\dot{x}_1 = -a_{11}x_1 + f_1(a_{22} - jx_3)x_2 + f_1 u, \tag{6}$$

$$\dot{x}_2 = a_{21}x_1 - (a_{22} - jx_3)x_2, \tag{7}$$

$$\dot{x}_3 = 0, \tag{8}$$

$$y = h(x) = x_1. \tag{9}$$

3. Observability conditions of the complex model

In this section the observability property of the dynamic model (6)–(9) is investigated. As is well known, the model in question is locally weakly observable if the observability matrix, of order $\infty \times n$, given by

$$O = \begin{pmatrix} dh(x) \\ dL_f h(x) \\ dL_f^2 h(x) \\ \vdots \end{pmatrix},$$

where $dh(x)$ is the gradient of h at x, $L_f h(x)$ is the Lie derivative of h along f, $L_f^k h(x) = L_f L_f^{k-1} h(x)$, and

$$f(x,u) = \begin{pmatrix} -a_{11}x_1 + f_1(a_{22} - jx_3)x_2 + f_1 u, \\ a_{21}x_1 - (a_{22} - jx_3)x_2, \\ 0 \end{pmatrix},$$

has rank equal to n.

That being stated, the following theorem gives necessary and sufficient conditions for the observability of the model (6)–(9).

Theorem 1. *Suppose that stator current measures, $y(t) = x_1(t)$ are available over an infinite time period $t \in [0, \infty)$. Then, the IM model*