Control Engineering Practice 53 (2016) 57–67

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Experimentally verified generalized KYP Lemma based iterative learning control design ☆

Wojciech Paszke [a,*], Eric Rogers [b], Krzysztof Gałkowski [a]

[a] University of Zielona Góra, ul. Szafrana 2, 65-516 Zielona Góra, Poland
[b] Department of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, United Kingdom

ARTICLE INFO

Article history:
Received 27 August 2015
Received in revised form
14 January 2016
Accepted 17 April 2016
Available online 11 May 2016

Keywords:
Iterative learning control
Finite frequency range design
The generalized KYP lemma
Experimental verification

ABSTRACT

This paper considers iterative learning control law design for plants modeled by discrete linear dynamics using repetitive process stability theory. The resulting one step linear matrix inequality based design produces a stabilizing feedback controller in the time domain and a feedforward controller that guarantees convergence in the trial-to-trial domain. Additionally, application of the generalized Kalman–Yakubovich–Popov (KYP) lemma allows a direct treatment of differing finite frequency range performance specifications. The results are also extended to plants with relative degree greater than unity. To support the algorithm development, the results from an experimental implementation are given, where the performance requirements include specifications over various finite frequency ranges.

1. Introduction

Iterative learning control (ILC) has been especially developed for systems defined over a finite duration that repeat the same task. Each execution is known as a trial, or pass, and the sequence of operations is that a trial is completed, the system resets to the starting location and then the next trial begins, either immediately after the resetting is complete or a further period of time has elapsed. The novel feature of this control law design method is the use information from the previous trial, or a finite number of previous trials, to update the control input applied on the next trial and thereby improve performance from trial-to-trial. In particular, the control objective is to find an input such that the corresponding output precisely tracks a reference signal that is specified over a finite time interval.

Since the original work, widely credited to Arimoto, Kawamura, and Miyazaki (1984), ILC has remained as a significant area of control systems research with, especially for linear model based designs, many algorithms experimentally verified in the research laboratory and applied in industrial and other applications. An overview of developments up to their dates of publication can be found in, e.g., the survey papers (Ahn, Chen, & Moore, 2007; Bristow, Tharayil, & Alleyne, 2006; Wang, Gao, & Doyle, 2009), where the last of these has a particular focus on run-to-run control as found in the chemical process industries. Applications areas include industrial robotics, see, e.g., Longman (2000) and Norrlöf (2002), where the pick and place operation common in many mass manufacturing processes is an immediate fit to ILC, and wafer stage motion systems, see, for example, Heertjes and Tso (2007). More recent applications areas include flexible valve actuation for non-throttled engine load control (Heinzen, Gillella, & Sun, 2011) and various forms of industrial printing, see, for example, Barton, Mishra, Alleyene, Ferreira, and Rogers (2011), Bolder (2015) and large dynamic range nanoprinting (Parmar, Barton, & Awtar, 2014). Also there has been a transfer from engineering to next generation healthcare for robotic-assisted upper limb stroke rehabilitation with supporting clinical trials (Freeman et al., 2009, Freeman, Rogers, Hughes, Burridge, & Meadmore, 2012).

One common approach, see, for example, Ahn et al. (2007) and Bristow et al. (2006) as starting points for the literature, to ILC design is to first apply a feedback control law to stabilize and/or produce acceptable along the trial dynamics and then apply ILC to force trial-to-trial error convergence of the resulting system. This is a two step design approach with separate design of the feedback and learning filters where, for example, the ILC learning update is calculated as the inverse of the dynamics resulting from the feedback controller design. Moreover, the ILC learning update is a feedforward signal from the previous trial and hence does not affect the stability property of the dynamics along the trials, that is, as the trial duration is finite, trial-to-trial error convergence can

☆This work is partially supported by National Science Centre in Poland, grant No. 2014/15/B/ST7/03208.
* Corresponding author.
E-mail addresses: w.paszke@issi.uz.zgora.pl (W. Paszke), etar@ecs.soton.ac.uk (E. Rogers), k.galkowski@issi.uz.zgora.pl (K. Gałkowski).

http://dx.doi.org/10.1016/j.conengprac.2016.04.011
0967-0661/

occur for unstable dynamics but the result could be unsatisfactory/unacceptable along the trial dynamics. For discrete dynamics using the lifting approach can result in the need for computation with very large dimensioned matrices.

An alternative approach to ILC design is to use the 2D systems setting, that is, systems that propagate information in two independent directions where for ILC these directions are from trial-to-trial and along each trial respectively. Early work in this setting used the Roesser state-space model for 2D discrete linear systems (Kurek & Zaremba, 1993; Paszke & van de Molengraft, 2007). Repetitive processes (Rogers, Gałkowski, & Owens, 2007) are a distinct class of 2D systems where information in the temporal domain is limited to a finite duration and hence a more natural match to ILC. These processes make a series of sweeps, termed passes or trials in the ILC setting, through a set of dynamics defined over a finite duration and once each pass is complete the process resets to the starting location. On each pass an output, termed the pass profile, is produced that acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. The result can be oscillations that increase in amplitude from pass-to-pass.

Repetitive processes cannot be controlled by direct application of standard systems theory and algorithms and this has led to the development of a stability theory for them and substantial progress on control system specification and design (Rogers et al., 2007). More recent research has used the repetitive process setting to design ILC laws with experimental verification (Hładowski et al., 2010; Paszke, Rogers, Gałkowski, & Cai, 2013). The result is a one step design for trial-to-trial error convergence and transient response along the trials and hence simultaneous treatment of the trial-to-trial error and transient response along the trials is possible. These results are not, however, completely compatible with practical requirements, because they use state feedback control and therefore experimental implementation or a physical application requires that all entries in the current trial state vector are available for measurement, or a state estimator is used, in addition to the requirement that these measurements are not noise corrupted. Also, a static gain used as the learning filter does not enable satisfactory performance in some cases since the same gain is used for entire frequency range.

The new design developed in this paper considers simultaneous synthesis of both feedback and learning controllers in an ILC scheme for error convergence and performance, starting with a new result for monotonic trial-to-trial error convergence. This result is achieved by converting the problem to one of the stability along the trial for a discrete linear repetitive process, leading to design based on Linear Matrix Inequality (LMI) computations. Also the benefits of this new result relative to existing alternatives are highlighted.

As in other areas for linear systems theory and design, it is necessary to design for stability and performance, where for the latter aspect the requirements for each case must be formulated into design constraints. Such specifications can include regulating against the effects of exogenous signals, penalizing regulated variables and specifying the level of plant uncertainty allowed. In this paper it is established that, as in other areas of standard linear systems analysis and design, such requirements can be expressed as conditions on the maximum singular value of the frequency response matrix coupling the errors on successive trials.

One method, again as in standard case, would be to introduce weighting filters to emphasize a particular frequency range, followed by design to ensure that weighted system norm is sufficiently small. In this paper, the generalized KYP lemma is used to develop a new design method where weighting filters are not required and hence an unnecessary increase in the controller order is avoided. In addition, an equivalence between a frequency domain inequality and LMIs over finite frequency range is established. This does not require the use of a constant frequency independent Lyapunov matrix over the entire frequency range and hence the design conservatism is reduced in comparison to alternative solutions. To support the analysis and design, results from experimental application to an electromechanical system are given. Another contribution of the paper is the development of a method that deals with the case when the system to be controlled has relative degree greater than unity through the use of an anticipative feedforward control law.

The design of ILC for linear systems with higher relative degree has been the subject of considerable attention, where the results currently available in the literature are based on the assumption that the learning controller is a static gain and the first Markov parameter of controlled system (i.e., CB from the state-space triple $\{A;B;C\}$) defining the system model is known. However, the first Markov parameter is the response at time step one for a unit pulse input at step zero and hence, in typical situations, this parameter has zero (or near to zero) value. Therefore the solution can generate a very high gain and can result in very poor performance and transient dynamics. An alternative solution should include the fact that the desired trajectory must start from the first time step for which an input at zero can influence the output. This paper develops a systematic way to design both the learning and feedback controllers for systems with higher relative degree. It is shown that application of anticipative control law is possible and some transformations lead to a problem form where not all matrices of the resulting state-space model are changed.

One alternative way of designing ILC laws for strictly proper systems is given in Hładowski et al. (2011) but this design is based on state vector augmentation. Also, the dimensions of the matrix variables grow as the relative degree of the system increases. As a result, the controller matrix dimensions are also increased. The approach developed in this paper avoids these problems and the controller matrix dimensions do not increase with the plant relative degree. Also the dimensions of the matrix variables does not increase and therefore reduces the computational load when compared to the approach in Hładowski et al. (2011).

Another approach was given in Paszke, Gałkowski, and Rogers (2012) where a low-pass filter with unity DC gain and sufficiently high cut-off frequency was used to remove a system limitation for the relative degree one case only. However, in practise it is hard to determine a high cut-off frequency prior to design procedure and also choosing the order of the filter. Finally, this approach is applied when simple structure static learning and feedback controllers are used. These are significant limiting factors in the application of this approach that are overcome in this paper.

Throughout this paper, the null and identity matrices with the required dimensions are denoted by 0 and I, respectively, and the notation $X \prec Y$ (respectively $X \succ Y$) means that the matrix $X - Y$ is negative definite (respectively, positive definite). Also $\mathrm{sym}\{M\}$ is used to denote the symmetric matrix $M + M^{T}$ and $\rho(\cdot)$ denotes the spectral radius of its matrix argument, i.e., if λ_i, $1 \le i \le q$, denotes the eigenvalues of a $q \times q$ matrix, say H, $\rho(H) = \max_{1 \le i \le q} |\lambda_i|$. The superscript $*$ denotes the complex conjugate transpose of a matrix and $\otimes$ the matrix Kronecker product.

We make use of the following results, where the first is the generalized KYP lemma and the second the Elimination (or Projection) Lemma.

Lemma 1 (*Iwasaki and Hara, 2005*). *Consider matrices* $\mathbb{A}$, $\mathbb{B}_0$, Θ *and*

$$\Phi = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \Psi = \begin{bmatrix} 0 & e^{j\omega_c} \\ e^{-j\omega_c} & -2\cos(\omega_d) \end{bmatrix}, \tag{1}$$

with $\omega_c = (\omega_l + \omega_u)/2$, $\omega_d = (\omega_u - \omega_l)/2$ *and* ω_l, ω_u *satisfying* $-\pi \le \omega_l \le \omega_u \le \pi$. *Suppose also that* $\det(e^{j\omega}I - \mathbb{A}) \ne 0$ *for all*