Control Engineering Practice 51 (2016) 81–91

Contents lists available at ScienceDirect

Control Engineering Practice

journal homepage: www.elsevier.com/locate/conengprac

Guidance using bearing-only measurements with three beacons in the plane

Minh Hoang Trinh [a], Gwi-Han Ko [a], Viet Hoang Pham [a], Kwang-Kyo Oh [b], Hyo-Sung Ahn [a,*]

[a] *School of Mechatronics, Gwangju Institute of Science and Technology (GIST), 1 Oryong-dong, Buk-gu, Gwangju 500-712, South Korea*
[b] *Automotive Components and Materials R& D Group, Korea Institute of Industrial Technology, Gwangju, South Korea*

ARTICLE INFO

Article history:
Received 13 August 2015
Received in revised form
29 February 2016
Accepted 18 March 2016
Available online 4 April 2016

Keywords:
Navigation
Guidance
Bearing-only measurements (BOM)
UAV

ABSTRACT

This paper proposes a bearing-only measurement based guidance algorithm for a mobile agent to navigate in a two-dimensional plane. Based on the bearing vectors and the subtended angles related to three stationary beacons, the proposed algorithm allows the agent to reach its desired location. We show that the agent reaches its desired location globally asymptotically based on Lyapunov stability theory. Under some assumptions, the agent is also proved to reach the desired location exponentially based on Lyapunov's indirect method. Simulations and actual experimental tests on quadrotor system are also provided to verify the effectiveness of the proposed algorithm in outdoor environment.

1. Introduction

Recently, small-sized unmanned aerial vehicles (UAVs) have attracted a significant amount of interest (Sullivan, 2006) from researchers in the control system society. Due to their mobility, UAVs are useful in a variety of applications including surveillance, search and rescue, assessing disaster areas, etc. (Bouabdallah, Becker, & Siegwart, 2007). A fundamental task of the UAVs is autonomous navigation, which allows them to reach a desired location without human interaction. In order to achieve the navigation task, global positioning system (GPS) is usually used for outdoor operation. However, in many scenarios, such as indoor operation, urban cityscapes or rugged mountainous terrain, GPS information is either unavailable or unreliable. Hence, it is important to design an effective navigation strategy for UAVs.

In the literature, we can find several navigation strategies based on simultaneous localization and mapping (SLAM) (Dissanayake, Newman, Clark, Durrant-Whyte, & Csorba, 2011). Using measurements corresponding to some stationary beacons, an autonomous agent can localize its relative position with respect to the beacons and decide its navigation strategy accordingly (Madhavan & Durrant-Whyte, 2004). For example, SLAM strategies based on miniature laser sensor (Aghamohammadi, Tamjidi, & Taghirad, 2008; Grzonka, Grisetti, & Burgard, 2012), optical flow sensor (Kendoula, Fantoni, & Nonami, 2009), or embedded camera (Courbon, Mezouar, Guenard, & Martinet, 2010; Huili Yu & Randy Beard, 2010) have been extensively studied. Generally, distances and bearing angles are both estimated from agent's sensors. The agent, then, maintains a local coordinate frame, where relative positions of the beacons are all represented.

The goal of this work is to design a bearing-only measurement (BOM) based guidance algorithm for an agent in the plane. There are several motivations for this work. Firstly, BOM based algorithms permit us to reduce the number of equipped sensors of UAVs for navigation task. Secondly, one could reduce the overall system cost by using BOM based algorithms when designing a multi-agent system consisting of a large number of UAVs, e.g., the formation control problems (Oh, Park, & Ahn, 2015). Finally, BOM based algorithms are useful in radio silent applications (Deghat, Shames, Anderson, & Yu, 2014). The bearings can be obtained from image sensors (Choi & Kim, 2014), thus no signal has to be transmitted when the UAVs are on maneuvers.

For related works, BOM based tracking and guidance problems have been extensively studied in the literature. Considering a slowly moving target in a 2D plane, the authors in Deghat, Xia, Anderson, and Hong (2016) proposed a BOM based algorithm that simultaneously estimates the location of a moving target and circumnavigates around it. Navigation based on some stationary beacons were exploited, for examples, in Argyros, Bekris, and Kavraki (2004) and Loizou and Kumar (2007). Assuming that there are three noncolinear stationary beacons, the authors in Argyros et al. (2004) introduced a moving-along-the-bisector strategy for a mobile robot to reach almost all location in the entire plane. In a

* Corresponding author.
E-mail address: hyosung@gist.ac.kr (H.-S. Ahn).

http://dx.doi.org/10.1016/j.conengprac.2016.03.013
0967-0661/© 2016 Published by Elsevier Ltd.

similar setup, the authors in Loizou and Kumar (2007) developed a BOM based control law for navigating and tracking several moving targets in the plane. Under their control law, the agent moves until all desired bearing vectors are achieved, and its velocity is determined from the projection of desired bearing vectors into the orthogonal complementary space of the measured bearing vectors. A disadvantage of this control law is the requirement of a global coordinate, which is usually unavailable in autonomous navigation. In Trinh, Lee, and Ahn (2015), a BOM based control law was developed to guide an agent to the geometric median of N beacons in n-dimensional space, thus, minimizing the weighted distance sum from the agent to the beacons. BOM based control laws also attracted attentions in formation control problems, in which a group of agents cooperates to form a desired geometric shape (Oh et al., 2015). Bearings can be obtained from an embedded camera, thus it motivates the so-called vision based formation control (Das et al., 2002; Moshtagh, Michael, Jadbabaie, & Daniilidis, 2009). In Basiri, Bishop, and Jensfelt (2010) and Bishop (2011), the authors solved the bearing-only formation control problem in three and four agents' cases using only local bearing angles. They proved that desired formation shapes can be globally asymptotically achieved if each agent controls the subtended bearing angle between its two adjacent agents. A BOM based control law stabilizing cyclic formations with an arbitrarily number of agents in the plane was also provided in Zhao, Lin, Peng, Chen, and Lee (2014). In that work, the cyclic formations can be exponentially or finite-time stabilized by tuning a parameter in the control law. Besides, collision avoidance was also considered in their works (Zhao et al., 2014). However, bearing-only based formation control has a limitation in controlling the formation shape. In fact, the bearing-based formation scale depends on the initial positions of the agents. To control the formation scale, atleast a distance variable must be controlled (Bishop, Summers, & Anderson, 2013) or there are several leaders capable of controlling their positions (Trinh, Oh, & Ahn, 2014a, 2016; Zhao & Zelazo, 2015a,b).

In this paper, we provide a novel BOM based guidance algorithm in the plane. Assuming that there are three stationary beacons on the plane, we design the guidance algorithm for a mobile agent based on the local bearing vectors and subtended bearing angles, which allows the agent to determine its relative position and reach its desired location. Based on the Lyapunov stability theory, we show that the agent asymptotically reaches the desired position under the proposed guidance algorithm. The analysis in this paper is motivated from several related works in bearing only formation control and network localization (Basiri et al., 2010; Bishop & Basiri, 2010; Bishop et al., 2009). Note that in the conference version of this paper (Trinh, Oh, & Ahn, 2014b), the control law is only valid inside the triangular region formed by the beacons. Thus, this paper extends the previous control law to the entire plane. Further, simulation and experimental results with quadrotor systems are presented to test the algorithm in outdoor environment.

The rest of the paper is organized as follows. In Section 2, we formulate a BOM based guidance problem. In Section 3, a BOM based control law to steer an agent to reach any desired location inside the triangle formed by three beacons will be introduced and analyzed. From that control law, in Section 4, we further propose an algorithm to navigate in the entire plane. Simulation and experiment results will be provided in Section 5. Finally, conclusion and suggestions for further studies are provided in Section 6.

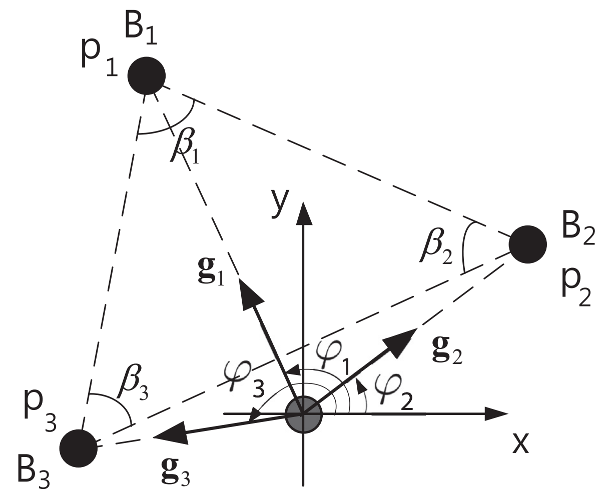

Fig. 1. The bearings φ_i and the bearing vectors $\mathbf{g}_i$, $i = 1, 2, 3$.

2. Problem statement

Consider a mobile agents modeled by the following single-integrator:

$$\dot{\mathbf{p}} = \mathbf{u}, \tag{1}$$

where $\mathbf{p} \in \mathbb{R}^2$ and $\mathbf{u} \in \mathbb{R}^2$ are the position and the control input at time instance t, respectively, with respect to a global Cartesian coordinate system. Although the control law is practically implemented in the agent's local coordinate system, in this paper, since the local and the global representations of (1) are proved to be equivalent, we will only use the global representation in the following analysis.

We assume that there are three beacons $\mathbf{B}_1$, $\mathbf{B}_2$, and $\mathbf{B}_3$, which are located at $\mathbf{p}_1$, $\mathbf{p}_2$, and $\mathbf{p}_3$ on the plane, and further the positions of the beacons are non-collinear. We also assume that the initial position of the agent is different from beacons' position, i.e. that $\mathbf{p}(0) \neq \mathbf{p}_i$, $i \in \{1, 2, 3\}$. Moreover, suppose that the agent can only obtain the bearing angles with regard to the beacons. In the two-dimensional space, the bearing from an agent to a beacon $\mathbf{p}_i$ can be defined as the counter clockwise angle φ_i $(0 \le \varphi_i < 2\pi)$ between the x-axis of the coordinate frame and the line segment connecting $\mathbf{p}$ and $\mathbf{p}_i$ as shown in Fig. 1. From bearing measurement, the agent can construct a corresponding bearing vector (Trinh et al., 2015; Loizou & Kumar, 2007),

$$\mathbf{g}_i = \frac{\mathbf{p}_i - \mathbf{p}}{\| \mathbf{p}_i - \mathbf{p} \|} = \mathbf{1}\angle\varphi_i, \tag{2}$$

where $i \in \{1, 2, 3\}$ and $\mathbf{1}$ is the unit vector in the x-axis. Note that the bearing vector defined in (2) contains direction information toward the beacon and it does not require any distance information.

Moreover, the algebraic dependence between three available bearings implies the information about the agent's relative position with regard to the three beacons. For $i = 1, 2, 3$, let $\vartheta_i = |\varphi_{(i-1)} - \varphi_{(i+1)}|$, where $i - 1 = 3$ if $i=1$, and $i + 1 = 1$ if $i=3$ for convenience. Then $0 \le \vartheta_i < 2\pi$. We define the subtended bearing angle as (Bishop & Basiri, 2010)

$$\alpha_i = \begin{cases} \vartheta_i & \text{if} \quad 0 \le \vartheta_i \le \pi, \\ 2\pi - \vartheta_i & \text{if} \quad \pi < \vartheta_i \le 2\pi, \end{cases} \tag{3}$$