Electric Power Systems Research 119 (2015) 364–369

Contents lists available at ScienceDirect

Electric Power Systems Research

journal homepage: www.elsevier.com/locate/epsr

On continuous numerical Fourier transform for transient analysis of lightning current related phenomena

Slavko Vujević, Dino Lovrić*

University of Split, Faculty of Electrical Engineering, Mechanical Engineering and Naval Architecture, R. Boškovića 32, 21000 Split, Croatia

ARTICLE INFO

Article history:
Received 20 March 2014
Received in revised form 15 October 2014
Accepted 18 October 2014
Available online 7 November 2014

Keywords:
Fourier transform
Frequency domain
Finite element technique
Adaptive recursive sampling
Lightning current

ABSTRACT

In this paper a novel numerical method is proposed for solving the Fourier transform of an arbitrary transient function. Fourier integral is solved using a combination of numerical and analytical integration resulting in a continuous transient function in the frequency domain. The proposed numerical algorithm enables robust and accurate transformation of any arbitrary transient function into the frequency domain. This also includes arbitrarily sampled measured lightning currents. The result of the transformation in the frequency domain is continuous and completely independent of the time domain sampling procedure unlike in transformation algorithms that are generally employed for this purpose. Accuracy of the proposed algorithm has been verified on a number of numerical examples.

1. Introduction

Numerical models in transient analysis of electromagnetic phenomena can be generally classified as time domain models and frequency domain models depending on the domain in which the main computation occurs [1]. In most cases transient analysis refers to lightning strike analysis although overvoltages in electrical systems can occur due to switching loads. One of the main problems of the frequency domain approach is the transformation of the transient function describing source voltage or current from the time domain into the frequency domain [2]. Naturally, this is only the case if the function cannot be analytically integrated into the frequency domain. The second problem with the frequency domain approach is the subsequent transformation of the solution function(s) from the frequency domain into the time domain. This is mainly due to the fact that most transformations used in these models are discrete transforms and the sampling procedure in the frequency domain is dependent on the sampling in the time domain.

The problem of transforming the transient function from the time domain into the frequency domain is in most cases solved using the discrete Fast Fourier Transform (FFT) algorithm which is a tool primarily employed in signal processing analysis [3]. FFT algorithm is characterized by: (a) uniform sampling in the time and frequency domain, which is unsuitable for transient analysis, (b) mutual dependence of time and frequency samples, and (c) complex selection of the time window. Time window in the FFT algorithm has to be chosen in such a way that the lightning current function which is to be transformed into the frequency domain becomes insignificant at the end of this interval. Additionally the unknown functions which are to be transformed from the frequency domain into the time domain must also become insignificant at the end of this interval. The limitations of this algorithm when used in transient analysis are described in detail in [4].

In this paper an algorithm for the continuous numerical Fourier transform (CNFT) of an arbitrary transient function (with an emphasis on lightning current function) is proposed. Unlike the FFT algorithm, CNFT algorithm employs arbitrary sampling in the time domain which can be uniform or nonuniform. Furthermore, time domain samples do not in any way influence the consequent frequency domain samples due to the fact that using the CNFT one obtains a continuous function in the frequency domain. In the FFT algorithm this is not the case since discrete values at predefined frequency samples are obtained.

The proposed CNFT algorithm is based on arbitrary sampling in the time domain and linearization of the lightning current function over a single time segment (finite element), similar as in [5]. In other words, lightning current function is approximated using the finite element technique [6] with two-node finite elements. In the finite element technique terminology this implies that the global nodes are in fact sampling points. This linearized lightning current function can then be analytically transformed into the frequency domain. Due to the fact that only the lightning current function has been approximated and the oscillatory part of the Fourier integral

* Corresponding author. Tel.: +385 21305805; fax: +385 21463877.
E-mail addresses: vujevic@fesb.hr (S. Vujević), dlovric@fesb.hr (D. Lovrić).

http://dx.doi.org/10.1016/j.epsr.2014.10.020
0378-7796/

has been analytically integrated, highly accurate and robust results are obtained.

Using the proposed CNFT algorithm, one can also obtain a continuous transform of a measured lightning current which enables researchers to perform transient analysis with actual measured data. This can be accomplished accurately even if the measured values of the lightning current do not become insignificant in the observed interval.

2. Continuous numerical Fourier transform (CNFT)

An arbitrary lightning current in the time domain $i(t)$ can be transformed into the frequency domain using Fourier transform [6]:

$$\bar{I}(\omega) = \int_0^{+\infty} i(t) \cdot e^{-j \cdot \omega \cdot t} \cdot dt \tag{1}$$

where $\bar{I}(\omega)$ is the lightning current in the frequency domain, t is the time, $\omega = 2 \cdot \pi \cdot f$ is the circular frequency and f is the frequency.

Lightning current $i(t)$ can be modeled using a number of functions which include but are not limited to the double-exponential function [7], Heidler function [8], Heidler function for approximating lightning currents with two different rise portions [9], CIGRE function [10], Nucci function [11], Diendorfer and Uman function [12] and NCBC function proposed by Javor [13]. Only the double-exponential function can analytically be transformed into the frequency domain. Other functions such as Heidler function, Nucci function, Diendorfer and Uman function and NCBC function can be transformed using an analytical-numerical integration [13,14].

In this paper, a novel numerical algorithm for a continuous transformation of an arbitrary lightning current function from the time domain into the frequency domain is proposed. Using this CNFT algorithm it is possible not only to transform lightning current functions from the time domain into the frequency domain but also measured lightning currents. It is important to note that in the CNFT algorithm, unlike in the FFT algorithm, the discretization in the time domain does not influence the subsequent discretization in the frequency domain. CNFT algorithm is based on arbitrary discretization (uniform or nonuniform) of the lightning current function in the time domain and linearization of the lightning current function over a time segment (finite element). In other words, the lightning current is approximated using the finite element technique, more specifically using finite elements with two nodes (Fig. 1). Sampling points therefore represent global nodes of all finite elements.

Lightning current linearized over a single finite element can be analytically transformed into the frequency domain so, in the CNFT algorithm, Eq. (1) is transformed into the following equation:

$$\bar{I}_{CNFT} = \sum_{k=1}^{NEL} \int_{t_k}^{t_{k+1}} i_{FET}(t) \cdot e^{-j \cdot \omega \cdot t} \cdot dt \tag{2}$$

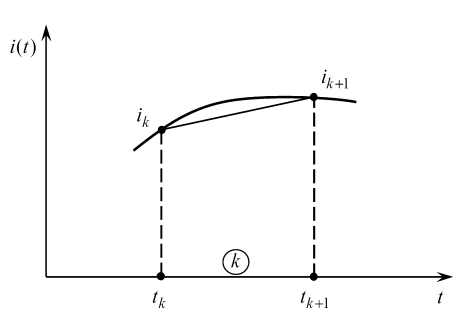

Fig. 1. Linear approximation of the lightning current in the time domain over the kth finite element.

where NEL represents the total number of finite elements over which the lightning current is linearized, t_k is the starting time of the kth finite element, t_{k+1} is the ending time of the kth finite element and $i_{FET}(t)$ is the linear approximation of the lightning current over the kth finite element. Linear approximation $i_{FET}(t)$ is given by the following expression:

$$i_{FET} = \frac{t_{k+1} - t}{t_{k+1} - t_k} \cdot i_k + \frac{t - t_k}{t_{k+1} - t_k} \cdot i_{k+1} \tag{3}$$

where $i_k = i(t_k)$ is the value of the lightning current in the kth global node and $i_{k+1} = i(t_{k+1})$ is the value of the lightning current in the global node $k+1$.

Introducing Eq. (3) into Eq. (2) and performing analytical integration, the following expression for lightning current in the frequency domain is obtained:

$$\bar{I}_{CNFT}(\omega) = \sum_{k=1}^{NEL} \frac{(i_{k+1} - i_k) \cdot \bar{A}_k + (t_{k+1} \cdot i_k - t_k \cdot i_{k+1}) \cdot \bar{B}_k}{t_{k+1} - t_k} \tag{4}$$

where for $\omega \neq 0$:

$$\bar{A}_k = \frac{e^{-j \cdot \omega \cdot t_{k+1}} - e^{-j \cdot \omega \cdot t_k}}{\omega^2} + j \cdot \frac{t_{k+1} \cdot e^{-j \cdot \omega \cdot t_{k+1}} - t_k \cdot e^{-j \cdot \omega \cdot t_k}}{\omega} \tag{5}$$

$$\bar{B}_k = j \cdot \frac{e^{-j \cdot \omega \cdot t_{k+1}} - e^{-j \cdot \omega \cdot t_k}}{\omega} \tag{6}$$

while for $\omega = 0$:

$$\bar{A}_k = \frac{t_{k+1}^2 - t_k^2}{2} \tag{7}$$

$$\bar{B}_k = t_{k+1} - t_k \tag{8}$$

The accuracy of the CNFT algorithm applied on lightning current functions depends on:

- Estimation of maximum time $T_{max} = t_{NEL+1}$, i.e. selection of the time window. The value of lightning current should become

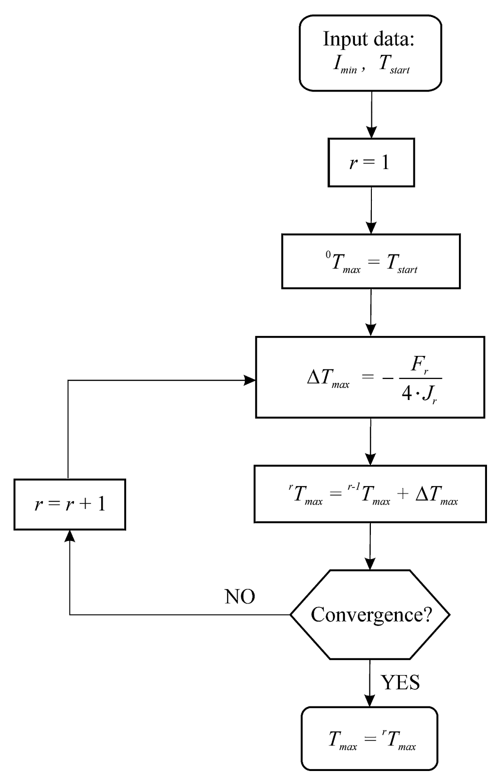

Fig. 2. Estimation of T_{max} using the Marquardt least squares method.