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Reformulation of parameters of the logistic function applied to power curves of wind turbines

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ABSTRACT

The current procedure for obtaining the parameters of the logistic function, used as a model for the power curve of wind turbines, provides meaningless values. These values are different for each wind turbine and obtaining them requires an optimization process. This paper proposes a procedure to obtain the parameters of the 4-parameter logistic function based on the features of the power curve, providing a model that is a function of the power curve parameters supplied by the manufacturer. Furthermore, that model can be used to derive another 4-parameter model and a 3-parameter model is proposed for certain conditions. The three models consist of a continuous function which simplifies the implementation of the curve in a computer program compared to piecewise models. In addition, the probability density function of the output power of a wind turbine is derived by using each model.

1. Introduction

The power curve of a wind turbine (WT) reveals the relationship between the wind speed at hub height and the electric power supplied, and so it is widely used when analyzing or studying a WT or a wind farm. Here, it is understood that the power curve of a WT is the one provided in the manufacturer's documentation. This paper will focus on this curve although measurements in WTs in a wind farm will eventually reveal operating conditions which can slightly differ from those described by the manufacturer. Of course, circumstances such as turbulence, wind shear, wake effects, icing, gusty winds or even component fatigue in the WT will affect the operating condition. As time goes by, other problems linked to ageing may also appear and these can interfere with optimum output of the WT blades. Power curves given by the manufacturer can nonetheless be interpreted as good approximations to a standard WT behaviour, so there will be no more discussion here about operating conditions themselves.

As defined in [1], WT manufacturers obtain the power curve from tests that provide pairs of points for wind speed-power every 0.5 m/s. Hence, the power curve will be the function, $P = f(u)$, which minimizes the distance to these points. Manufacturers [2–6] usually provide a graph establishing this relationship, which can be very helpful when obtaining the output power of a WT from the wind speed at hub height. However, when dealing with a large amount of data and in order to implement the relationship in a software program, the use of such a graph can be cumbersome, and this is why mathematical expressions are used for power curves. The expressions can substitute the pairs of points for piecewise continuous or continuous functions.

Among the applications of the models of the power curve it can be pointed out to indicate anomalies in the WT working, to forecast the power supplied by a WT, to simulate potential scenarios of wind power production and to compare the performance of different WT.

Currently, several models are preferred for establishing the relationship because they have a low value for errors and are easy to manage. In most cases, models consist of a continuous or piecewise function defined for all values of wind speed, extending the values obtained in the manufacturer's tests [7] to all possible values. Furthermore, given that the output power of a WT equals zero for values of wind speed lower than the cut-in wind speed, u_{ci}, (2–5 m/s) and higher than the cut-out one, u_{co}, (20–30 m/s), this will be the interval to define a model for the power curve.

The most commonly used models are those that utilize polynomial functions to approximate the relationship mentioned previously. Linear [8], quadratic [9–12], cubic [13–15], least-square [16,17] and spline [16,18–20] models are well known, but there are others [16,21–24]. In all cases the function is piecewise due to the specific shape of the power curve, which can be seen in Fig. 1. However, there are some other models that are based on a single continuous function [25–27] and provide good results. Among

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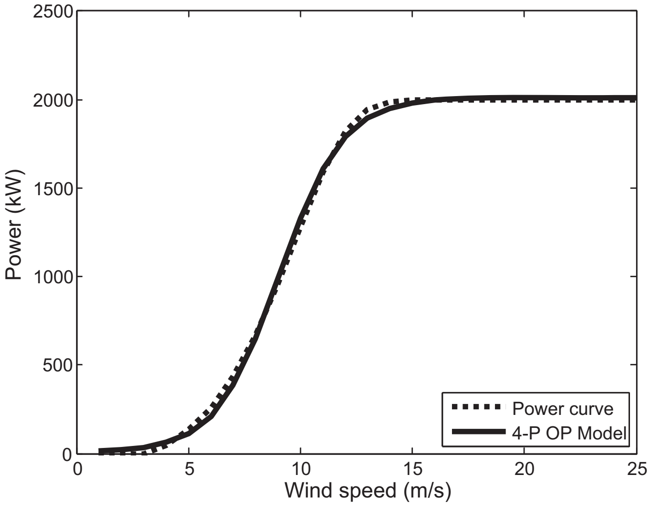

Fig. 1. 4-Parameter logistic model for the power curve of Vestas V80.

them, the 4-parameter logistic function and the 5-parameter logistic function are most commonly used.

Among the piecewise models, linear, quadratic and cubic ones appear most frequently in the literature. However, when compared with those based on a single continuous function, they provide errors around the rated wind speed of the WT that are almost completely avoided when using the logistic functions. In fact, in piecewise models, there is a lack of continuity of the slope around the rated wind speed value, which does not represent the real working of the WT, while in the case of the logistic functions this does not happen because they consist of a single function and it is derivable on the domain. This is the main reason of the inaccuracy of the piecewise models.

Moreover, when performing electrical system analysis it is very helpful to make available continuous models in order to derive analytical expressions that can be used as inputs of the analysis itself. For example, an analytical method to solve the Probabilistic Load Flow requires the Probability Density Function (PDF) of the power supplied by the generators as input, which can be easily obtained from the logistic functions and can hardly be derived in a single expression from piecewise models.

As all these models need the use of the aforementioned parameters, some procedure must be described for obtaining them. In fact, for the 4-parameter logistic function the procedure is already established [25] and the results of the comparison between the manufacturer's power curve and the model can be seen in Fig. 1.

The error values when using the 4-parameter (4-P) logistic model using an optimization process (OP) to obtain the parameters (4-P OP Model) for Vestas V80 and other WT are shown in Table 1 (Mean Absolute Percentage Error, MAPE), Table 2 (Mean Absolute Error, MAE) and Table 3 (Root Mean Squared Error, RMSE).

The procedure suggested in [25] may not always be the best option as this will depend on the kind of objective being pursued. In this paper, an alternative procedure to obtain the parameters of the 4-parameter logistic function is provided in order to improve the model. Additionally, some simplifications of the model are presented.

This paper is organized as follows: in Section 2, the 4-parameter logistic function is introduced and the suggested procedure for obtaining its parameters is given. In Section 3, the PDF of the output power of a WT is derived using the models presented here, taking advantage of its continuity. Section 4 includes results of the application of the models for several WTs and the conclusions are finally given in Section 5.

Table 1
MAPE values for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	0.0096	0.0100	0.0079	0.0078	0.0118	0.0087	0.0087
4P-DP	0.0162	0.0133	0.0151	0.0119	0.0169	0.0143	0.0129
4P-DS	0.0185	0.0103	0.0163	0.0113	0.0181	0.0148	0.0111
3P-DP	0.0198	0.0109	0.0176	0.0124	0.0194	0.0158	0.0117
Linear	0.0492	0.0474	0.0770	0.0394	0.0200	0.0694	0.0443
Quadratic	0.0902	0.0307	0.1324	0.0763	0.0299	0.1194	0.0763
Cubic	0.1276	0.0563	0.1775	0.1099	0.0599	0.1607	0.1099

Table 2
MAE values for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	19.2696	20.4170	18.1581	16.0462	27.1972	31.4132	60.5502
4P-DP	32.4614	27.2336	34.7689	24.3311	38.8846	51.3634	89.9470
4P-DS	36.9029	21.2036	37.5170	23.2441	41.6216	53.3197	77.3338
3P-DP	39.5474	22.3069	40.5822	25.3636	44.6043	57.077	81.6301
Linear	98.4040	97.2145	177.0171	80.8145	46.1067	249.6831	310.0873
Quadratic	180.3773	62.9964	304.6291	156.3958	68.7169	429.8800	533.5663
Cubic	255.2821	115.5071	408.2103	225.3624	128.5431	578.3840	768.8938

Table 3
RMSE values for several WTs using the models proposed.

	Vestas V80	Enercon E82	Siemens 82	Repower 82	Nordex N90	Siemens 107	Vestas V164
4P-OP	25.3319	26.2691	24.5692	21.2895	39.1501	42.6797	95.6218
4P-DP	56.2597	39.7432	59.9937	38.1073	68.3914	85.3121	146.1193
4P-DS	55.4600	35.1524	54.6059	35.5069	65.3848	80.6301	129.8385
3P-DP	60.1208	38.5687	60.3170	40.6524	72.2248	90.3829	147.0264
Linear	184.6801	172.6738	303.5940	156.1934	86.0137	449.0788	449.0788
Quadratic	313.4841	128.0031	486.2839	278.4891	143.2952	714.1565	714.1565
Cubic	431.7377	230.4151	639.4723	392.4452	247.3919	941.3501	941.3501