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Short communication

# Ringdown analysis of power systems using vector fitting

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ABSTRACT

This short communication introduces vector fitting for mode estimation from ringdown responses of power systems. The performance of the method is evaluated for different power system models and is found to be very accurate.



## 1. Introduction

Monitoring and detection of the oscillatory modes contained in a post-disturbance "ringdown" response can provide vital information for power system stability [1]. Mode estimation from ringdown responses can reflect directly, and almost in real-time, the dynamic characteristics of a system. Several techniques have been proposed to analyze online and offline ringdown responses. Among the most popular are the spectral analysis by means of Fourier transform [2], the Prony [3] and the matrix pencil [4] methods, as well as the Hilbert–Huang transform [5]. In 1999 vector fitting (VF) [6,7] was proposed, as a powerful and very accurate method for system identification in the frequency-domain (FD) and thus it has been adopted in many engineering areas, including high voltage power systems, microwave systems and high-speed electronics. VF proved to be robust and fast, providing very accurate fitting with guaranteed stable poles. In this short communication VF is implemented for the first time in the identification of the oscillatory modes of ringdown responses in power systems.

## 2. Ringdown analysis

A system ringdown response containing $N$ modes is described by:

$$y(t) = \sum_{i=1}^{N} \frac{1}{2} A_i e^{\pm j\varphi_i} e^{\lambda_i t} = \sum_{i=1}^{N} A_i e^{\sigma_i t} \cos(\omega_i t + \varphi_i) \quad (1)$$

where $\lambda_i = \sigma_i \pm j\omega_i$ stand for the system eigenvalues, $\omega_i = 2\pi f_i$ and $\sigma_i$ are the angular frequency and the damping factor of the $i$-th mode, and $A_i$, $\varphi_i$ are the corresponding amplitude and phase, respectively [1]. The Laplace transform $Y(s)$ of (1) is a rational function expressed by:

$$Y(s) = \sum_{i=1}^{N} \frac{c_i}{s - p_i} \quad (2)$$

where $p_i$ and $c_i$ are the poles and residues defined as:

$$c_i = \frac{A_i \times e^{\pm j\varphi_i}}{2}, \quad p_i = \lambda_i = \sigma_i \pm j\omega_i \quad (3)$$

Given a fixed number of samples for $Y(s)$ as resulted from applying the Fast Fourier Transform (FFT) to the discretized form of (1), VF can approximate (2) by means of a two-stage linear least squares problem [6,7]. At the first stage, VF relocates a set of initial poles to better positions by solving the linear equation of (4) with the known poles $\alpha_k^{(m)}$, where $m$ denotes the $m$-th iteration. Since the zeros $\tilde{z}_k^{(m)}$ of $\sigma'^{(m)}(s)$ approach the poles $p_i$ of $Y(s)$, they are calculated by solving the eigenvalue problem of (5) with $\tilde{d}^{(m)}$ and matrices $\boldsymbol{A}^{(m)}$,

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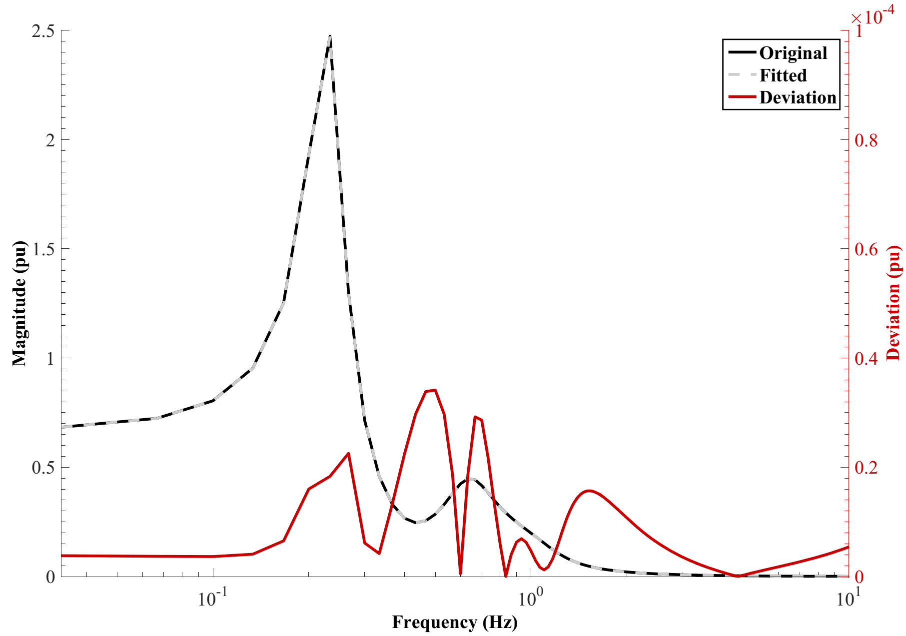


**Fig. 1.** Comparison of the original and fitted FD response. (For interpretation of the references to color in the text, the reader is referred to the web version of this article.)

$\boldsymbol{b}^{(m)}$ and $\boldsymbol{c}^{(m)}$ defined by the rational model of $\sigma'^{(m)}(s)$. By replacing the poles with the new ones, an improved set is achieved until $\alpha_k^{(m)}$ tends to $p_i$. At the second stage, the unknown residues are calculated by solving (4) with $\sigma'^{(m)}(s)$ equal to unity [6,7]. Finally, the mode parameters contained in the ringdown are calculated from the resulting poles using (3).

$$\underbrace{\left(\sum_{k=1}^{N}\frac{\tilde{r}_k^{(m)}}{s-\alpha_k^{(m)}}+\tilde{d}^{(m)}\right)}_{\sigma'^{(m)}(s)}Y(s)\cong\underbrace{\sum_{k=1}^{N}\frac{r_k^{(m)}}{s-\alpha_k^{(m)}}+r_0^{(m)}}_{\sigma'Y^{(m)}(s)} \quad (4)$$

$$\tilde{\boldsymbol{z}}^{(m)}=\boldsymbol{\alpha}^{(m+1)}=\mathrm{eig}\left(\boldsymbol{A}^{(m)}-\boldsymbol{b}^{(m)}\tilde{d}^{-1(m)}\boldsymbol{c}^{T(m)}\right) \quad (5)$$

## 3. Numerical results

### 3.1. Synthetic signal mode identification

The performance of VF for mode identification is evaluated using test signal 1 (TS1) of (6) [8]. TS1 is generated at a rate of 100 samples/s (sps) assuming total observation time of 30 s to simulate a PMU data stream [1,9]. The number of the identified poles is automatically adjusted, by reaching either the maximum number of poles or the relative error tolerance criteria. Possible additional artificial modes, which are mostly observed in cases of noise distorted signals, are surplus to the dominant modes. These modes can be easily detected and removed, since they are characterized by significantly low amplitude or high frequency. In the examined case, the spectrum of the ringdown is fitted by VF assuming a maximum of 10 poles and a relative error tolerance equal to −40 dB.

$$y_{TS1}(t)=\underbrace{1.0\times e^{-0.1697t}\cos(2\pi\times0.2284\times t-0.8\pi)}_{\text{mode\#1}}+\underbrace{1.32\times e^{-0.815t}\cos(2\pi\times0.625\times t+0.6\pi)}_{\text{mode\#2}}+\underbrace{1.13\times e^{-1.823t}\cos(2\pi\times1.029\times t+0.1\pi)}_{\text{mode\#3}} \quad (6)$$

The resulting FD magnitude of the original and the fitted spectrum are compared in Fig. 1. As shown in the right $y$ axis, depicted with red color, the absolute deviation is lower than 4.0E − 5 over the whole frequency range, while in Table 1 the relative prediction error (PE) is found to be very small for all mode parameters. The identified mode parameters are used to simulate the time-domain (TD) response in Fig. 2 and negligible differences are observed compared to the original ringdown response, since the coefficient of determination ($R^2$), defined in (7), is 99.97%.

**Table 1**
% PE of the identified mode parameters.

| | Mode #1 | Mode #2 | Mode #3 |
|---|---|---|---|
| $\sigma$ | 5.7E − 4 | 9.9E − 3 | 2.9E − 3 |
| $\omega$ | 3.1E − 4 | 2.9E − 3 | 1.3E − 2 |
| $A$ | 3.7E − 2 | 1.5E − 2 | 2.4E − 2 |
| $\varphi$ | 1.9E − 2 | 1.4E − 2 | 2.3E − 1 |

$$R^2=\left(1-\frac{\sum_{k=1}^{M}\left(y(k)-\hat{y}(k)\right)^2}{\sum_{k=1}^{M}\left(y(k)-\bar{y}\right)^2}\right)\times100, \quad (7)$$

where $y$ is the original signal response with mean value $\bar{y}$ and $\hat{y}$ is the corresponding signal estimate. A 100% value for $R^2$ reveals that the original signal is perfectly fitted, while a 0% value that the estimated signal is a constant ($\hat{y}=\bar{y}$).

Moreover, the computational burden of the proposed identification method is investigated using an Intel Core i7-4770, 3.4 GHz, RAM 8 GB personal computer. The resulting processing timings and $R^2$ are summarized in Table 2 for different sampling rates. It can be seen that the method can accurately fit the signal response and extract the dominant modes in all cases. The corresponding computational time is significantly low for sampling rates up to 100 sps that most PMUs support, revealing the feasibility of the proposed method for online applications.