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Natural convection and heat transfer on a section-triangular roof

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ABSTRACT

Natural convection and heat transfer on a section-triangular roof are common around buildings. In this study, the slope flow and the plume on the suddenly heated section-triangular roof are investigated using scaling analysis and numerical simulation. The dynamics and heat transfer are discussed. It has been demonstrated that there exist different regimes of transient natural convection on the roof, which depends on the Rayleigh number, the aspect ratio of the roof and the Prandtl number. The scaling laws in different scenarios including inertial and viscous regimes are obtained and verified by numerical results. There is agreement between the scaling laws and numerical results. Further, the formulae of heat transfer and natural convection on the roof are presented.

1. Introduction

Natural convection widely exists in buildings and plays a role in building environment and energy conservation [1–3]. In particular, natural convection and heat transfer on a roof have received considerable attention due to their significance [4–10].

Natural convection on the roof usually involves a slope flow along the roof and a plume above the roof. The slope flow on the roof is also termed as a thermal boundary layer flow on a heated inclined wall, which has been considerably investigated [11]. In the early study [12], a one-dimensional model of laminar natural convection on a heated or cooled inclined wall was presented, and the theoretical solution was obtained under the Boussinesq approximation. The temperature of the laminar slope flow was measured and in turn the local heat transfer was given in [13], which agrees well with that on a vertical wall if the inclination angle is large. Further, an analysis of the two-dimensional laminar slope flow was also performed and the analysis solution obtained is consistent with the experiment in [14].

The stability of the slope flow has also been considered due to its fundamental and practical significance [15]. The instability of the slope flow can be described by for example longitudinal rolls on the inclined wall, which were observed in [16]. Furthermore, it is found that perturbations in the slope flow are similar to those in the vertical wall experiment [17]. Additionally, the dependence of the stability of the slope flow on governing parameters has been studied in [18]. It is demonstrated that the critical distance where the slope flow starts to destabilize is dependent on the Prandtl number. The experiment shows that the separation occurs in the transitional slope flow and is also dependent on the inclination angle [19,20], which is consistent with the results of large eddy simulation [21]. The study [22] indicates that the slope flow may be turbulent if the Rayleigh number is sufficiently large (e.g., $Ra = 10^6$–10^9) based on direct numerical simulation. The slope flow can significantly enhance up to 32% of heat transfer in comparison with that on a horizontal heated wall. Further, the previous studies [14,19,23] show that the Nusselt number is proportional to $Ra^{1/5}$ for the laminar slope flow on an almost horizontal plate, to $Ra^{1/4}$ for the laminar slope flow [14] but to $Ra^{1/3}$ for the turbulent slope flow, and is also dependent on the inclination angle of the roof.

Recently, the dynamics of the transient slope flow have been focused [24]. The development of the slope flow can be classified into different stages. The scaling laws of the transient slope flow have been obtained. The effect of the Prandtl number on the dynamics of the transient slope flow has been discussed in [25]. Further, the ramp heating condition of the inclined wall has also been considered in [25,26] in which the scaling laws of the velocity and thickness of the slope flow under the ramp heating have been presented. Through the comparison between the ramp and steady time scales, it has been demonstrated that if the ramp time is larger than the steady time, the slope flow can reach a quasi-steady mode dominated by the balance between conduction and convection [27]. The above mentioned scaling laws have also been validated by numerical results.

After the separation of the slope flow, the plume appears above the roof [28]. Early studies focused on the mixture and diffusion of the fluid due to the entrainment from the ambient fluid to the plume [29,30]. Morton et al. [31] presented the classic theory for which a set of ordinary differential equations is used to quantify the entrainment.

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Nomenclature

A	aspect ratio of the roof
C_p	specific heat (j/kg/K)
g	acceleration due to gravity (m/s^2)
h	height of the roof (m)
l	half-length of the roof (m)
L	length of the inclined wall of the roof (m)
n	coordinate normal to the boundary
Nu	Nusselt number
p	pressure (kg/m/s^2)
P	non-dimensional pressure
Pr	Prandtl number
Q	flow rate (m^2/s)
Ra	Rayleigh number
t	time (s)
$t_{g\nu}$	time of the transition of the plume between under inertial and viscous regimes (s)
$t_{Pg\kappa}$	time of the transition of the plume between under inertial conduction and inertial convection regimes
$t_{P\nu\kappa}$	time of the transition of the plume between under viscous conduction and viscous conduction regimes
t_s	time when the thermal boundary layer becomes steady (s)
T	temperature (K)
T_0	initial temperature of the fluid (K)
u_T	velocity for the slope flow at an unsteady state (m/s)
u_{Ts}	velocity for the slope flow at a steady state (m/s)
U_ν	non-dimensional velocity for the slope flow under an unsteady viscous regime
v_g	velocity for the plume under an unsteady inertial regime (m/s)
v_{gs1}	velocity for the plume under a steady inertial regime (m/s)
$v_{P\nu}$	velocity for the plume under an unsteady viscous regime (m/s)
$v_{\nu s}$	velocity for the plume under a steady viscous regime (m/s)
V	non-dimensional velocity for the plume
V_ν	non-dimensional velocity for the plume under an unsteady viscous regime
$V_{\nu s}$	non-dimensional velocity for the plume under a steady viscous regime
x, y	horizontal and vertical coordinates (m)
X,Y	non-dimensional horizontal and vertical coordinates
β	coefficient of thermal expansion (1/K)
δ_{Pg}	thickness of the plume under an unsteady inertial regime (m)
δ_{Pgs1}	thickness of the plume under an steady inertial regime (m)
$\delta_{P\nu}$	thickness of the plume under an unsteady viscous regime (m)
$\delta_{P\nu s}$	thickness of the plume under a steady viscous regime (m)
δ_T	thickness of the thermal boundary layer (m)
δ_{Ts}	thickness of the thermal boundary layer at a steady state (m)
Θ	non-dimensional temperature
κ	thermal diffusivity (m^2/s)
ν	kinematic viscosity (m^2/s)
Ρ	density (kg/m^3)
Τ	non-dimensional time
τ_s	non-dimensional time at which the thermal boundary layer becomes steady
ΔT	temperature difference between the inclined wall and the fluid (K)
Δ_P	non-dimensional thickness of the plume
Δ_{Ts}	non-dimensional thickness of the thermal boundary layer when the thermal boundary layer becomes steady
Δτ	non-dimensional time step

However, the study [32] indicates that the classic theory based on the assumption of a virtual point heat source is limited for the plume on a finite-sized heat source. Therefore, a great number of theoretical, numerical and experimental studies of the plume rising from a point or a line heat source have been performed [33]. The classic theory has been extended and a starting plume has been focused [34]. The study in [35] shows that the development of a starting plume undergoes through four different ascent stages: a conduction stage, a cap velocity increase stage, a "plateau" stage where the cap velocity remains constant and a cap velocity decrease stage. That is, the fluid in the vicinity of the heat source is firstly heated following sudden heating, and conduction dominates natural convection around the heat source [36]. A starting plume appears since Rayleigh-Bénard instability occurs in the fluid around the heat source [37]. Subsequently, the starting plume develops and even breaks [38]. The velocity of the plume has been measured in [39], which is consistent with the scaling relations by [40] and asymptotic solution by [41]. Additionally, the plume on different heat sources has also been investigated [42] and a wide range of governing parameters has been considered (see e.g., [43]).

A literature review shows that the slope flow and the plume have been investigated. However, less attention was paid into complex natural convection on the roof. Particularly, the dynamic regimes of transient natural convection on the roof are still unclear and in turn the corresponding heat transfer needs to be quantified, which is the motivation of this study. In this study, transient natural convection and heat transfer on the roof are investigated. The dynamics and heat transfer of the slope flow and the plume are discussed using a simple scaling analysis. Different regimes and possible scenarios of the slope flow and the plume are identified, which are dependent on the three governing parameters: the aspect ratio of the roof, the Rayleigh number and the Prandtl number. The scaling laws of the velocity and thickness of the slope flow and the plume for different regimes and scenarios are obtained and validated by numerical results.

In the rest of this paper, the problem description and the scaling analysis are presented in Section 2; Numerical results and validation of the scaling laws are given in Section 2; and Section 4 summarizes conclusions.

2. Scaling analysis

Under consideration is natural convection on a section-triangular roof, as illustrated in Fig. 1. The height and half length of the roof are h and l, respectively. At the initial time, the roof is suddenly heated from T_0 to $T_0 + \Delta T$ where T_0 is the initial temperature of the environmental fluid. The development of transient natural convection on the roof is governed by the equations including the Navier-Stocks and temperature equations with the Boussinesq approximation,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \tag{2}$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta\Delta T, \tag{3}$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \tag{4}$$

After the normalization of Eqs. (1)–(4) (also see Eqs. (27)–(30)), it is