International Communications in Heat and Mass Transfer 92 (2018) 64–72

Contents lists available at ScienceDirect

International Communications in Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ichmt

Numerical modeling of multiple steady-state convective modes in a tilted porous medium heated from below

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ARTICLE INFO

Keywords:
2D numerical modeling
Porous medium
Free convection
Boussinesq approximation

ABSTRACT

Numerical simulations are carried out to determine the steady-state convective modes in a rectangular porous cavity heated from below. The property of multiplicity of solutions for a given set of governing parameters is examined in this paper. The multiple steady-state solutions that appear in a horizontal cavity for a given Rayleigh number are obtained by means of suitable initial conditions. Each of these solutions is then perturbed by increasing the inclination angle in order to identify the transition angle to a different convective mode. It is observed that for an odd-number of convective cells, if the counterclockwise rotating cells dominate the configuration, the Nusselt number increases with the slope angle up to a maximum and then decreases before the transition to single cell convection. Otherwise, if there are more clockwise rotating cells, the Nusselt number decreases monotonically and the configuration becomes unstable. Since multicellular configurations with even number of convective cells have equal number of clockwise and counterclockwise rotating cells, this case presents a single behavior characterized by a decrease in the Nusselt number. The transition angles from multicellular to single cell convection are found to be as large as 45° when the aspect ratio of the cavity is large, so that this angle is the upper limit to destabilize multicellular convection.

1. Introduction

Extensive research on free convection in porous media has been carried out in the past due to its importance in different scientific and engineering contexts. For instance, modeling of low enthalpy geothermal systems is a research area closely related with this topic [1, 2, 3]. In last years this problem has been addressed from a variety of perspectives. Baytaş and Pop [4] studied free convection in oblique porous enclosures. Baytaş [5] and Meshram [6] considered entropy generation effects in an inclined porous cavity. Khanafer [7] looked into non-Darcian effects in free convection in a porous medium. Bennacer et al. [8] conducted anisotropy studies in a vertical porous enclosure with double diffusive convection. Free convection in porous media in conditions of turbulent flow and mass transport has also been studied [9, 10]. de Lemos [11] incorporated non-thermal equilibrium conditions, and more recently Carvalho and de Lemos analyzed the case of free convection in laminar flow and mass transport assuming also non-thermal equilibrium conditions [12]. Although these works explore different aspects of the physics that govern free convection in porous media, some questions regarding the steady state convective modes that arise in 2D in such systems are still unanswered. For instance, whereas multicellular and single cell convective modes are well known, the different forms multicellular convection can adopt has not been reported in detail. The transition angles of these convective modes to single cell have not been reported either. The numerical study presented here determines the multiple steady-state convective modes that exist in such a system and their transition angles. At the same time this work aims to provide a wider perspective for the analysis of steady-sate free convection in porous media in three-dimensions (3D) [13, 14].

Fundamental aspects of the problem analyzed here are given by the solution of the Horton-Rogers-Lapwood problem [15]. This problem establishes the conditions for the onset of convection in a horizontal porous layer heated from below. The early works by Horton and Rogers [16] and Lapwood [17] determined a critical Rayleigh number ($Ra_c = 4\pi^2$) for the onset of convection in such a system. Elder [18] presented one of the first numerical and experimental studies of steady-state convection for this problem. He described the steady state cellular motion of the fluid, incorporating edge-effects of the porous enclosure. Straus [19] conducted stability analysis and reported that as the Rayleigh number increases the wavenumber of the system increases, and

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https://doi.org/10.1016/j.icheatmasstransfer.2018.02.009

Nomenclature

Greek symbols

α	Slope angle
β	Thermal expansion coefficient
η	Overall thermal diffusivity
μ	Viscosity
ψ	Stream function
ρ	Density
σ	Ratio of solid-fluid heat capacities
θ	Dimensionless temperature

Other symbols

–	Overbar denotes dimensional variable

Roman letters

$\mathbf{u}$	Darcy's velocity
$\hat{A}$	Amplitud
B	Characteristic length
C	Length
D	Aspect ratio
g	Gravitational constant
k	Permeability
L_∞	Infinity norm
n	Number of convective cells
Nu	Nusselt number
P	Pressure
Ra	Darcy-Rayleigh number
T	Temperature
t	Time
x, y, z	Cartesian coordinates

Subscripts

0	Reference quantity
c	Critical quantity
int	Simulation time interval
l	Local
ss	Steady state
t	Transition

for $Ra \geq 380$ there are no stable 2D solutions. Likewise, De La Torre Juárez and Busse [20] showed that the maximum Nusselt number of steady state convection corresponds to higher wavenumbers as Ra increases.

Kaneko et al. [21] carried out an experimental study of free convection in an inclined porous enclosure. They found that there is an angle at which the system reaches the maximum level of convective motion, characterized by multiple convective cells. They also reported that above this angle the system evolves towards single cell convection. This transition between multicellular and single cell convection was addressed numerically by Moya et al. [22]. They analyzed the change of the steady state solutions as the slope angle and Rayleigh number were varied. Their model successfully reproduced the appearance of single-cell convection as it was shown experimentally by Bories and Combarnous [23], however due to the steady-state numerical scheme they were only able to obtain a limited number of multicellular convective modes. This arises the question that what are the possible multicellular configurations before reaching single-cell convection. The existence of multiplicity of steady state solutions was described by Sen et al. [24], who reported that multiple steady states exist when the inclinations with respect to the heated wall are small enough and some of which are unstable. Riley and Winters [25] described the mechanisms through which the multiple solutions reduce to leave an apparently unique solution for large slope angles. Rees and Bassom [26] found the maximum inclination angle at which multicellular convection can become unstable, which is $\alpha = 31.49^\circ$ corresponding to a critical Rayleigh number of 104.30. In consistency with this result, Báez and Nicolás [27] calculated transition angles for a Rayleigh number of 100 and different aspect ratios of the porous cavity. Likewise, they observed different multicellular configurations with different number of cells for a given set of governing parameters.

The different multicellular steady state solutions that can exist in a sloping porous enclosure heated from below are found here by inducing the system to host arbitrary numbers of convective cells. This is done by means of providing suitable initial conditions of the governing equations. Each of the induced multicellular configurations will become a steady-state solution as long as such convective mode is stable. Additionally, the evolution of each of these multicellular configurations towards single cell convection is examined by increasing the slope angle in small steps until the solutions destabilize. Unlike previous studies, this work covers a detailed parametric space regarding the slope angle and the number of convective cells in the cavity, which accounts for more than twenty three thousand simulations. The numerical scheme was developed based on the stream function approach, which has been widely applied for the solution of free convection in both porous media and homogeneous fluids [28].

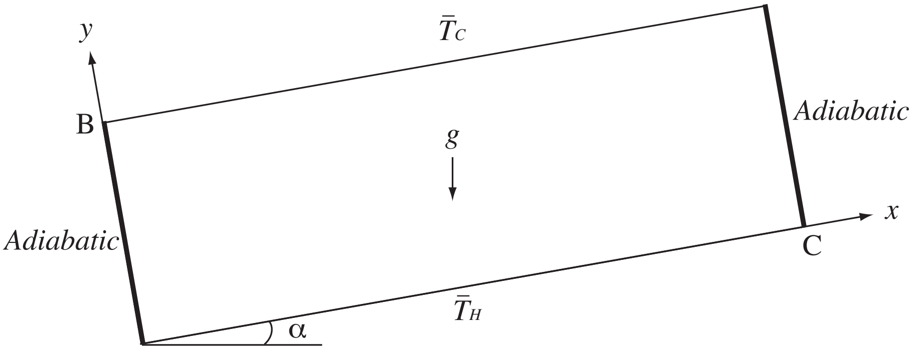

Fig. 1. Schematic model of a rectangular porous enclosure tilted an angle α.