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Wall-roughness eddy viscosity for Reynolds-averaged closures

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ABSTRACT

An approach to modeling the effect of rough surfaces on turbulent boundary-layer flow is proposed and developed. It is based on the concept of a rough-wall eddy viscosity, in which the pressure and viscous drag forces which arise on account of flow past roughness elements are recast as an equivalent viscous shear force within the roughness sublayer at the surface. This shear force is modeled as the wall-roughness eddy viscosity and carries information on both the flow Reynolds number and the roughness height. The modeling approach is developed and evaluated as part of a k–ε closure. When the wall-roughness eddy viscosity is modeled in proportion to $(k_s^+)^{3/4}$, where $k_s^+ = k_s u_\tau/\nu$ and is the sand-grain roughness in wall units, it yields predictions of drag coefficients which are in excellent agreement with those from reference data for flow in rough-walled pipes, over a wide range of surface-roughness heights and Reynolds numbers. Its predictions are also in good agreement with experimental data for zero and favorable pressure gradient boundary layers over fully-rough surfaces.

1. Introduction

The modeling of the mean turbulent flow over rough surfaces using Reynolds-averaged closures is a challenging problem with many practical applications. It has been pursued sporadically for the last half century, with most modeling approaches developed to try and match correlations of Nikuradse's experimental measurements of mean velocity profiles in pipe flows, the surfaces of which were coated with sand grains of a prescribed size (Cebeci and Bradshaw, 1977). In smooth, flat-wall boundary layers, the tangential stress exerted by the fluid on the wall is the viscous shear stress $\mu\partial\bar{u}/\partial y$. However, in rough-wall boundary layers, this shear stress is both modified by the flow and supplemented by the form drag per unit area, caused by the flow-induced pressure distribution around each roughness element, and the combination of these effects is to be modeled.

One of the earliest roughness-modeling proposals was that of Rotta (1962) who introduced the concept of an additive shift to the surface-normal coordinate employed in models ($y \to y + \Delta y$) to accommodate effects of surface roughness. This proposal was incorporated in the van Driest mixing-length model as

$$\ell^+ = \kappa(y^+ + \Delta y^+)\left\{1 - \exp\left[-\frac{y^+ + \Delta y^+}{A^+}\right]\right\} \quad (1)$$

with the spatial shift Δy modeled as the empirical function of the equivalent sand grain roughness which, after integrating the x-momentum equation, resulted in fair agreement with Nikuradse's velocity-profile data.

Yang et al. (2016) recently re-proposed a model based on the von Karman–Pohlhausen polynomial velocity profile method, in which the mean velocity in the roughness sublayer is modeled as an analytical function which grows exponentially with surface-normal distance, damped by a single prescribed attenuation coefficient that carries information on the character of the (rectangular-prism) surface roughness. Predictions made with this model, with the appropriate attenuation coefficient, were in good agreement with data for 'k' and 'd' type roughnesses on this surface, suggesting that analytical profile models might constitute a credible approach to modeling mean features of rough-wall boundary layers.

In the wall-function approach to modeling flow over rough walls, proposed for use with a k–ε model, differential equations for the flow variables are solved only in an outer region beyond a match point. The behavior of $\bar{u}$, k and ε between the wall and the match point is described by algebraic wall functions, the outermost values of which provide boundary conditions for the outer-flow k–ε computation. The wall functions are typically chosen to describe some aspects of local equilibrium in near-wall turbulence while including other features calibrated empirically to match computed mean velocity profiles to their measured counterparts at corresponding sand-grain roughness sizes (Patel and Yoon, 1995). Suga et al. (2006) have developed analytical wall functions for use with finite-volume discretizations which employ functional forms of equivalent sand-grain roughness to model the thickness of the viscous sublayer within the cell adjacent to the wall.

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They yielded relationships and model coefficients for the bulk effects of roughness on momentum and thermal energy within that cell. Predictions made with these wall functions in parallel flows, flat-plate and curved-wall boundary layers, and separating flows were in good agreement with the Moody chart and with experimental data.

An alternative to this approach is to replace the wall functions with a one-equation closure for k and a wall-function model for ε, together with lengthscales in the near-wall eddy viscosity model and ε function which incorporate the effect of surface roughness (Chen and Patel, 1988) or an equivalent one-equation modification (Aupoix and Spalart, 2003). Durbin et al. (2001) have demonstrated the performance of a two-layer model for wall roughness, which also included an additive shift in the wall-normal coordinate for turbulent quantities, for flows over ramps, sand dunes, and flat walls at different pressure gradients. For surfaces with roughness textures suited to modeling the very-near-wall flow as if through a porous medium, Liou and Lu (2009) have explored the use of a two-layer model with flow in the wall region described by the Brinkman equation, as a generalized Darcy's law, with an effective porosity and permeability. Their results compared favorably with experimental measurements of flow over smooth- and rough-walled airfoils.

A useful feature of the k–ω model (Wilcox, 1993) is that the effect of surface roughness can be modeled by a change in the wall-boundary value of ω ($= k/\varepsilon$), which yields solutions to the coupled k, ω and x-momentum equations which are in good agreement with the roughness-induced changes in mean velocity profile reported by Nikuradse. The wall value of ω can be generalized from its single smooth-wall value to an empirical function of sand-grain roughness, chosen to best match the targeted $\bar{u}$ profiles. If it is recognized that the ω (and ε) model equations serve primarily to set the timescale T as $1/\omega$ (or k/ε) in the eddy viscosity model $\nu_t = \text{const.}\, kT$, a change in the wall value of ω is equivalent to a change in the very-near-wall behavior of eddy viscosity ν_T and implicitly in $-\overline{u'v'}$ and $\bar{u}$. This boundary-value change and the consequent change in the near-wall behavior of the k and ω model equations appears to be sufficient to yield the experimentally observed shape of the $\bar{u}$ profile for a given value of the wall sand-grain roughness, without the need for additional modeling. The equivalent approach of modeling effects of surface roughness in the k–ε turbulence closure by changing only the boundary value of a variable does not appear to have been explored.

In the studies cited above, the surface topography is described as an equivalent sand-grain roughness height, and so is represented by a single correlating function or parameter. Schultz and Flack (2009) and Flack and Schultz (2010) have reviewed some of the shortcomings of this approach, namely the difficulty in finding measures of the geometric roughness height for which surface friction data will collapse in both the fully-developed and transitionally-rough regions. Their studies emphasized the importance of both roughness texture and geometric height in determining effects of roughness on mean velocity fields, which led them to propose that: (*i*) the average roughness height; (*ii*) higher-order statistics of height distribution; and (*iii*) a measure of the streamwise gradient of roughness height be used for general characterization of rough surfaces.

Recent applications of direct numerical simulation to turbulent flow over rough surfaces, with no-slip/no-penetration boundary conditions enforced at fine-grained roughness levels using immersed-boundary methods, which resolve flow within the roughness sublayer, have resulted in rough-wall boundary-layer data which are much more detailed than have previously been available (Yuan and Piomelli, 2014; Busse et al., 2015; Chan et al., 2015). Spatial decompositions and double-averaging procedures have also been developed to identify the 'extra' dispersive stresses and viscous and pressure drag forces which arise on account of surface roughness (Mignon et al., 2009). These advances have led to new physical insights into flow adjacent to rough surfaces and consequently renewed interest in developing engineering models of flows over rough walls.

In this paper, we describe an approach to modeling rough-wall boundary-layer flows which is based on insights into flow within the roughness sublayer from direct numerical simulations of flow over a surface with a single roughness texture, but different roughness heights. The terms in the x-momentum equation which account for pressure and viscous drag on account of roughness, as revealed by a double-averaging procedure, are modeled as an additive eddy-viscosity to mimic their role within the roughness sublayer so that it might be blended with a k–ε model in the outer flow, where this closure has more fidelity. The model is calibrated and tested against reference data for turbulent flow in rough-walled pipes over a range of roughness heights and Reynolds numbers in the transitionally-rough region, and over fully-rough surfaces beneath boundary layers in zero and favorable pressure gradients. Thus it is a step towards meeting the broader challenge of developing models for turbulent flow over surfaces with roughness textures of a more general nature.

2. Stress balance in rough-wall channel flow

In flows over rough walls, the turbulent boundary layer may be characterized by an outer region beyond a roughness sublayer, which extends from the trough to approximately the peak of the largest roughness element. Within the roughness sublayer, *intrinsic* ($\langle\rangle$, fluid variable averaged per unit fluid area or volume) and *superficial* ($\langle\rangle_s$, fluid variable averaged per unit total area or volume) averaging methods are used in conjunction with a time-space decomposition to define components of flow variables. Any representative flow variable θ can then be decomposed into: *(i)* the time-space average (the spatial $\langle\rangle$ average of a time average); *(ii)* the spatial variation about the time-space average ~; and *(iii)* the time-unsteady fluctuating component ′ as $\theta(x, y, z, t) = \langle\bar{\theta}\rangle(y) + \tilde{\theta}(x, y, z) + \theta'(x, y, z, t)$ (Mignon et al., 2009).

Beyond the roughness crest, shown in Fig. 2 at height k_c, the averaging region comprises only fluid, and intrinsic and superficial averages are identical. For the purposes of extending far-field turbulence closures and computations to the rough wall, flow variables at locations below the roughness crest are expressed as their Reynolds averages based on superficial spatial averaging. Thus the roughness geometry is incorporated in the rough-wall turbulence closure and can be solved on a smooth-wall computational grid. The location of the roughness trough is the x-axis, where boundary conditions are applied to the superficially averaged flow variables $\langle\bar{u}\rangle_s$ and $\langle\bar{k}\rangle_s$ (equivalent to $\bar{u}$ and k beyond the roughness sublayer), which are set to zero.

The time-space average of the x-momentum equation in steady, turbulent, constant-property fully-developed channel flow can be expressed (Yuan and Piomelli, 2014) in terms of superficially averaged variables and integrated with respect to y across a channel of half-width h, with $y = 0$ at the lower wall, to yield the stress balance across the channel:

$$-\rho(\langle\tilde{u}\tilde{v}\rangle_s + \langle\overline{u'v'}\rangle_s) + \mu\frac{\partial\langle\bar{u}\rangle_s}{\partial y} - \rho\int_y^h (f_p + f_v)dy = -\frac{\partial\langle\bar{p}\rangle_s}{\partial x}[h - y] \tag{2}$$

where $-(h/\rho)\partial\langle\bar{p}\rangle_s/\partial x$ is equal to the square of the mean friction velocity u_τ and the terms f_p and f_v are approximated as $-(1/\rho)\langle\partial\tilde{p}/\partial x\rangle_s$ and $\nu\langle\nabla^2\tilde{u}\rangle_s$ respectively, describing surface integrals over roughness elements of the pressure dispersion and viscous stresses on account of roughness (Raupach and Shaw, 1982).

In their simulations, Yuan and Piomelli (2014) found that the form-induced shear stress (the first term on the left of Eq. (2)) was small compared to the mean Reynolds stress (the second term from the left). They also found that, when expressed in wall units, the mean Reynolds stress $\langle\overline{u'v'}\rangle_s$ in rough-wall boundary layers was almost identical to its smooth-wall counterpart above the roughness crest, but lower in magnitude below it. Thus the effect of roughness within the roughness sublayer on the overall shear stress is primarily a reduction in the