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Solution of an inverse transient heat conduction problem in a part of a complex domain

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ABSTRACT

The purpose of this work is to formulate two simple methods which can be used to solve nonlinear inverse heat conduction problems in a part of a complex-shaped component in the on-line mode. The proposed methods can be useful if temperature measurements are carried out only in a selected part of an outsize element or if the whole element cannot be analysed due to the inverse heat conduction problem being ill-conditioned. It is assumed that the conductive heat transfer occurs through the surfaces separating the domain from the rest of the component. It is shown that the simplifying assumption of thermal insulation on these surfaces, which is often presented in literature, can cause significant errors. Compared to works published previously, if two additional unknown boundary conditions are introduced on the separating surfaces, the inverse problem conditioning deteriorates substantially. Despite this, stable solutions are achieved for "noisy measured data".

The presented methods can be used to optimize the power unit start-up and shutdown operation. They may also enable a reduction in the heat loss arising during the process and extend the power unit life. The methods presented herein can be applied in monitoring systems working both in conventional and nuclear power plants.

1. Introduction

Many structures in various technical applications operate under strong and often extreme thermal conditions. The steady-state or transient heat transfer phenomena can create non-uniform temperature fields. High thermal stresses often result in substantial temperature differences which should not exceed the allowable limit. The fundamental problem of calculating the temperature and stress distributions in operated elements is the difficulty in determining some of the thermal boundary conditions.

In the power unit components, an unknown boundary condition usually occurs on internal surfaces in contact with the fluid. In order to define this convection boundary condition, it is necessary to determine the heat transfer coefficient and the fluid temperature close to the element internal surface. The measurement of the two quantities is difficult because they can vary over time and space. However, they can be calculated analysing the phenomena that take place in the flowing fluid numerically by means of the control volume finite element method [1,2]. The whole area where the fluid is contained has to be discretized. Then the mass, momentum and energy balance equations have to be written. Depending on the flow nature, it may also be necessary to introduce a suitable turbulence model or two-phase flow models.

Another way to determine the distribution of temperatures is finding a solution of the inverse heat conduction problem (IHCP) in the device under analysis [3]. Inverse methods enable determination of the entire time- and space-dependent temperature distribution in an element based on measured temperature histories in selected spatial points. The solution can be found even though some thermal boundary conditions remain unknown. For elements with simple and regular shapes, assuming that the material thermal and physical properties are constant, exact methods of solving the inverse heat conduction problem may be used [4–6]. The methods mentioned above are fast-convergence procedures that have a non-iterative nature and are very effective in the case of one-dimensional problems. But for two- or three-dimensional ones, finding a solution is very laborious [6].

Solving inverse problems related to the heat exchange in bodies with complex shapes or with temperature-dependent thermal properties requires numerical methods. There are numerous studies devoted to one-dimensional problems [7,8]. The inverse method for solving steady-state inverse heat conduction problems in a furnace wall is presented in [9]. There are also many works devoted to two-dimensional problems. In [10], a two-dimensional

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linear transition inverse heat conduction problem is solved by means of the Generalized Minimal Residual Method in a quenching process using water jets. Three-dimensional ill-posed boundary inverse problems are solved by means of an iterative regularization method in [11]. A general method for solving multidimensional inverse heat conduction problems is presented in [12]. A new algorithm for identification of the refractory lining state and the heat transfer coefficient between the FCC zeolite catalyst particles and the regenerator walls can be found in [13]. A dedicated three-dimensional numerical model for inverse determination of the heat flux and the heat transfer coefficient distributions over the metal plate surface cooled by water is presented in [14]. However, all the methods mentioned above can be used only for elements with a simple geometry. An on-line inverse method combined with the finite-element scheme is proposed to make an inverse estimate of the unknown heat flux on the nozzle throat-insert inner contour in [15]. The finite-element scheme can easily define the irregularly shaped boundary. The formulation of an inverse method which can be used to solve inverse heat conduction problems in thick-walled complex-shaped elements is shown in [16]. The presented methods do not make it possible to solve the inverse problem in entire components but only in their parts. As the inverse solution is difficult to stabilize, large models cannot be built. It is hard to solve ill-posed problems because calculated temperatures are very sensitive to errors made while calculating "measured" temperatures or performing real-time measurements. The errors can create temperature oscillations, which can be the cause of an unstable solution. In order to overcome such difficulties, a variety of techniques have been proposed in literature, including regularization [17], future time steps [18], smoothing digital filters [16] and the domain decomposition method [18]. In [18], the space domain is divided into several sub-domains while assuming that both boundaries perpendicular to the unknown boundary (Γ_d) are insulated. The same approximation can be found in the inverse methods intended for simple [10–14] and complex geometries [15,16]. It should be emphasized that such an assumption is not always possible.

The purpose of this work is to formulate simple methods which can be used to solve nonlinear inverse heat conduction problems in a part of a complex-shaped component in the on-line mode. Heat may flow between the modelled part and the rest of the component through appropriate separating surfaces. Conducting the analysis in a smaller domain, it is possible to achieve better stabilization of the inverse method. The proposed procedure belongs to the group of space marching methods. It starts in a spatial node where the temperature sensor is located and marches through space sequentially to the surface with an unknown boundary.

2. Formulation of the problem

The three-dimensional heat conduction analysis can often be simplified to a two-dimensional conduction problem.

The equation governing the transient heat conduction problem given in [19] is as follows:

$$c(T)\rho(T)\frac{\partial T}{\partial t} = -\nabla \cdot \mathbf{q} + q_V, \tag{1}$$

where $\mathbf{q}$ is the heat flux vector defined by the Fourier law and q_V is the heat generation rate per a unit volume,

$$\mathbf{q} = -\mathbf{D}\,\nabla T. \tag{2}$$

$\mathbf{D}$ is the conductivity matrix and in a two-dimensional case it is defined as:

$$\mathbf{D} = \begin{bmatrix} k_x(T) & 0 \\ 0 & k_y(T) \end{bmatrix} \tag{3}$$

For isotropic materials $k_x(T) = k_y(T) = k(T)$. All material properties (c – specific heat, ρ – density, k – thermal conductivity) are assumed as known functions of temperature. The control volume finite element method is used to solve problems that occur in elements with complex geometries [20]. Eq. (1) is integrated over a general control volume V with a bounding surface S:

$$\int_V c(T)\rho(T)\frac{\partial T}{\partial t}dV = -\int_V \nabla\cdot\mathbf{q}\,dV + \int_V q_V\,dV \tag{4}$$

Control volume V is a volume with thickness H along the axis normal to plane xy (cf. Fig. 1).

By applying the mean value theorem for integrals on the left and the divergence theorem on the right, the following equation is constructed:

$$Vc(\overline{T})\rho(\overline{T})\frac{d\overline{T}}{dt} = -\int_S \mathbf{q}\cdot\mathbf{n}\,dS + \bar{q}_V V \tag{5}$$

where the bar indicates an average value in volume V. The product of the mean values is not equal to the average of the product, but for a small control volume V the error resulting from this approximation is small.

The domain of interest is the complex domain part limited by the surfaces shown in Fig. 1. S_{known} is the surface with a known boundary condition. In general, it will be defined by the second-kind boundary condition, and in the particular case it will be simplified to thermal insulation. $S_{1\ unknown}$ is the surface with an unknown boundary condition of any kind. $S_{2\ unknown}$ and $S_{3\ unknown}$ are the surfaces separating the analysed domain from the rest of the component and heat can flow through them by conduction. The methods for solving the IHCP in bodies with complex shapes which are presented in literature [15,16] assume only perfect insulation on the surfaces.

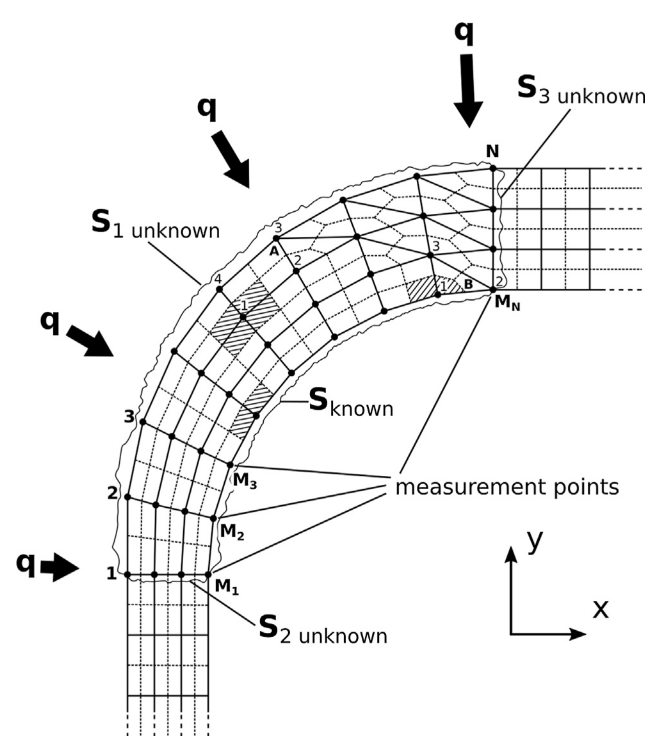

Fig. 1. Discretization of an irregularly shaped domain. S_{known} – surface with a known boundary condition; $S_{1\ unknown}$, $S_{2\ unknown}$, $S_{3\ unknown}$ – surfaces with unknown boundary conditions; M_1 to M_N – points of temperature measurement.