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A novel fractal model for permeability of damaged tree-like branching networks

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ABSTRACT

Damage of the tree-like branching networks in nature is a very prevalent phenomenon, which has attracted a great deal of attention in various fields ranging from animate to inanimate flow systems for several decades. In this study, a novel model for permeability is proposed for damaged tree-like branching networks based on the fractal characteristics of tree-like branching networks. It is found that the proposed model is a function of structure parameters of damaged tree-like branching networks, such as the numbers (p) of damaged channels, the diameter ratio (β), length ratio (α), branching number (n), branching level (k), and the total number of branching levels (m) in the network. The influence of the micro-structural parameters on the effective permeability of the networks is systematically studied. It is found that the numbers of damaged channels and branching levels have significant effect on its permeability and pressure drop. The present results are much different from the heat transfer in damaged tree-like branching networks. The model may have the potential in analysis and design of fluid flow systems.

1. Introduction

Tree-like branching networks, such as blood vessels, cooling systems in microelectronic, cracks in reservoir rocks and even traffic and social networks, abound in nature and man-made fluid flow systems. Lots of researchers have studied flow properties in the networks/systems due to many advantages in heat transfer and fluid flow [1–5] in the past decades.

The mechanism for heat transfer, fluid flow as well as gas diffusion in tree-like branching networks is an interesting theme in science and engineering applications. Murray et al. [6,7] may be the first investigator who systematically analyzed the blood flow property in cardiovascular system, and proposed a relation called the Murray's law for revealing a general physical principle in both living and nonliving transport networks. Generally, the law can be stated as that"the cube of the radius of a parent vessel equals the sum of the cubes of the radii of the daughter vessels."The optimal relationship between parent and daughter branch diameters was steadily supported by the experimental data and theoretical study [8,9]. Later, the optimal relationship, corresponding to structure parameters such as cross-sectional area, length and branching angle, was presented as well by the optimum biological principle [10]. Bejan [11] proposed the theoretical method, which is named as the Constructal law, in terms of thermodynamic optimization principle. It states: "for a finite-size flow system to persist in time (to live), its configuration must change in time such that it provides easier and easier access to currents that flow through it.", and "the Constructal law is the law of physics that accounts for the natural tendency of all flow systems (animate and inanimate) to change into configurations that offer progressively greater flow access over time." [12,13]. We found that the Constructal law covers all flow systems ranging from heat and fluid flow in nature to animal design, technology evolution, as well as social organization. Lorente and Bejan [14] designed a multiscale porous structure consisting of tree-shaped networks based on constructal law and found that the flow resistances of these architectures are lower than those of porous medium with the same volume and internal duct flow volume. Wattez and Lorente [15] designed the most probable pore path configurations by constructal approach, and this can minimize the diffusion transfer resistance and obtain the macroscopic diffusion coefficient. Their studies showed that the tree-like branching network is an optimal configuration for heat transfer and fluid flow [11–17].

In addition, tree-like branching networks in nature have self-similar fractal characters, and the structures may be adapted to various circumstances for survival because of their minimal energy dissipation [18,19]. Study of heat transfer and fluid flow in porous

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media and tree-like branching network has special advantages by means of fractal geometry [2,3,20–25]. Xu et al. [26] investigated the hydraulic conductivity of fractal tree-like branching network. They found that the fractal-like tree networks has much higher flow efficiency than that of the parallel channel network. Li and Yu [27] obtained the analytical solution of the starting pressure gradient for Bingham fluid in fractal tree-like branching network. Wang and Yu [28] studied the flow characteristics of the wetting and non-wetting phase power-law fluids in the fractal-like tree network. Later, Wang and Yu [29] derived the starting pressure gradient of Bingham fluids in the dual-porosity media embedded with fractal-like tree networks. They obtained the fractal models with no empiric constant, and in these models every parameter has clear physical meaning. Kou et al. [30] studied the optimal flow resistance in fractal tree-like branching networks with smooth and rough pipes for laminar flow and turbulent flow. They found that optimal diameter scale for laminar flow follows Murray's law. Luo et al. [31] developed an asymmetrical tree-like branching networks and found that its total flow resistance for laminar flow is much less than symmetric network with the same branching numbers and the terminal radii. Recently, Xu et al. [32] made a review on transport properties for heat transfer and fluid flow in tree-like networks. However, the fractal tree-like branching networks mentioned above are not damaged. Transport performance for heat transfer and fluid in fractal tree-like branching network may not be symmetric due to damage. Miao et al. [33] studied optimal structure of thermal conductivity in damaged tree-like branching networks. Their results show that the numbers of damaged channels and branching levels play a crucial role in the optimal diameter ratio as well as optimal thermal conductivity. However, the influence on fluid flow properties in fractal tree-like branching network with damaged structures is rarely studied to our best knowledge.

Note that the fractal tree-like branching network in this work is not in loops. Many kinds of tree networks with loops exit in nature, for instance, spider tree network and disc lotus leaf tree. Here we focus on the effect of the damaged fractal tree-like branching network without loops on fluid flow behavior in the networks. In the present work, we systematically study the influence of structure parameters of the fractal tree-like branching networks on the seepage behavior and pressure drop because of damage. A dimensionless permeability of the damaged tree-like branching networks is derived, and the relation between the permeability with the damaged branching number is discussed as well. The results might provide a better understanding of fluid flow mechanism in the damaged tree-like branching networks and may provide great potential for analysis and design of fluid transport systems with damaged structures.

2. The characteristics of fractal tree-like branching networks

The tree-like branching network with self-similar and fractal characteristics has special advantages for heat transfer and fluid flow [2,3,29–33]. In this work, we consider the tree-like branching network of Y-shaped as shown in Fig. 1(a), which is repeated by a finite number of the elemental branch as shown in Fig. 1(b). Here, we assume that every branch of the tree network is cylindrical tube. For the sake of simplicity, the pore-solid interface roughness and wall thickness of the tube are ignored. Every channel is divided into n branches at the next branching level (e.g. $n = 2$ in Fig. 1(a)), and the total number of branching levels is m in the whole network. Let length and diameter of a typical branch at some intermediate branching level k ($k = 0, 1, 2, 3 \ldots, m$) respectively be l_k and d_k.

For the fractal tree-like branching network, the two scale factors, which are independent of k, are defined respectively, i.e. length ratio $\alpha = l_{k+1}/l_k$ and diameter ratio $\beta = d_{k+1}/d_k$. Then, we have

$$l_k = l_0\alpha^k = l_m\alpha^{k-m} \tag{1}$$

$$d_k = d_0\beta^k = d_m\beta^{k-m} \tag{2}$$

where l_0 and d_0 are the length and diameter of the 0^{th} branching level, respectively.

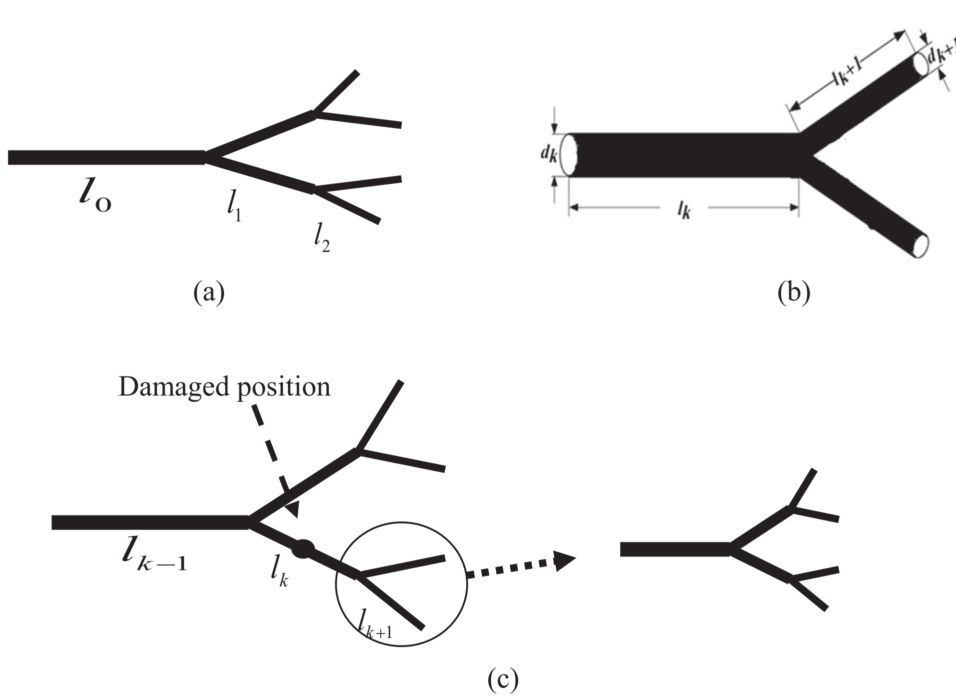

Fig. 1. (a) a schematic of fractal tree-like branching network, (b) an element structure (the k^{th} branching level), and (c) the damaged tree-like branching network in k^{th} branching level. The network in circle is fault.