International Journal of Heat and Mass Transfer 127 (2018) 362–372

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

# Inverse conduction and advection in a flat channel with transient external thermal excitation and observation

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ARTICLE INFO

*Article history:*
Received 19 February 2018
Received in revised form 10 May 2018
Accepted 29 May 2018

*Keywords:*
Conduction and advection
Conjugate heat transfer
Minichannel
Unsteady heat transfer
Convolution and deconvolution
Inverse problems
Measurement and instrumentation

ABSTRACT

The transient profiles of temperature and normal heat flux inside a flat minichannel heated by a surface heat source are constructed from temperature measurement over its external heated face. It uses analytical expressions of the corresponding transfer functions which are calculated using Laplace and Fourier integral transforms. Firstly, this estimation technique is verified on synthetic outputs of a finite elements code (COMSOL). Then it is implemented on an experimental minifluidic bench with electrical heating and temperature measurement by thermocouples and infrared thermography, for a low Péclet number of the flow. The presented results show that the heat source can be recovered at any time, as well as the internal normal heat flux and temperature distributions, including the bulk temperature of the liquid flow.



## 1. Introduction

In our previous works, we have shown the interest of transfer functions for linear time invariant systems where heat diffusion and advection occur, both on a theoretical [1] and on an experimental [2] basis. In this paper, which deals with a flat mini heat extractor, we will show how the surface heat source as well as the internal state variables (temperature and heat flux) can be estimated from temperature measurements over one of the outer faces using the corresponding transfer function. This non-destructive estimation technique, that corresponds to the construction of a virtual sensor, allows us therefore to estimate, in steady and transient regimes, the thermal state at locations difficult to access using direct measurements at another easy to access location. This requires the system to be linear with a geometry, thermophysical properties and fluid velocities that do not vary with time.

So, the topic dealt with in this paper derives from the now classical Inverse Heat Conduction Problem (IHCP) introduced by J.V. Beck et al. in the 80s [3] which consists in reconstructing surface temperatures or heat fluxes at part of the boundary of a solid domain, using known boundary conditions over its complementary part, the missing information being replaced by internal temperature measurements. This type of problem is mathematically ill-posed because the presence of noise in the data tends to make reconstruction of temperature or flux at the unknown part of the boundary unstable. This requires some special class of data processing called "regularization". This IHCP approach is very useful to estimate experimentally the distributions of both heat flux and temperature over the inner surface of the heated wall of a channel, using the diffusion heat equation in its solid volume, in order to derive the profiles of the internal convection coefficients. It can be used, for example, to optimize internal fluid mechanics in such a channel, see [4].

In Inverse Forced Convection Problems (IFCP) [5], the problem at stake is exactly the same, but the studied domain is a flowing fluid whose velocity field is known. The first works about this type of problems appeared in the 1990s and concerned estimation of inlet space [6] or time [7] temperature distributions in a flat heated channel [6] or the wall heat flux estimation in a flat [5,8] or annular [9] channel in transient thermal regime.

The precise subject of our work is the Inverse Conjugate Forced Convection Problem (ICFCP): the type of heat equation to be inverted is still the forced convection heat equation but it concerns not only a fluid, but also a solid subdomain, where a zero velocity field prevails, with the specific character that no heat transfer

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https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.142
0017-9310/

**Nomenclature**

| | |
|---|---|
| $\ell$ | channel length, m |
| $\mathcal{H}$ | Heaviside function |
| $h$ | heat transfer coefficient, W·m$^{-2}$·K$^{-1}$ |
| $L$ | virtual channel length, m |
| $Pe$ | Péclet number |
| $Re$ | Reynolds number |
| $T$ | temperature, K |
| $T_\infty$ | ambient temperature, K |
| $U_m$ | mean velocity, m·s$^{-1}$ |
| $x$, $y$ | spatial coordinates, m |
| $a$ | thermal diffusivity, m$^2$·s$^{-1}$ |
| $\mathcal{H}$ | transfer function |
| $p$ | Laplace parameter, s$^{-1}$ |
| $W$ | transmittance |
| $y$ | output, consequence, response |
| $Z$ | impedance |

*Greek symbols*

| | |
|---|---|
| $\alpha_n$ | discrete eigenvalue of order $n$ |
| $\lambda$ | thermal conductivity, W·m$^{-1}$·K$^{-1}$ |
| $\nu$ | kinematic viscosity, m$^2$·s$^{-1}$ |
| $\Phi$ | heat flow rate (surface integral of $\varphi$), W |
| $\rho$ | density, kg·m$^{-3}$ |
| $\varphi$ | heat flux density in $y$ direction, W·m$^{-2}$ |

*superscripts*

| | |
|---|---|
| $-$ | Laplace transform |
| $\sim$ | Fourier transform |
| $ss$ | steady state |
| $*$ | transposed of a matrix |

*subscripts*

| | |
|---|---|
| $f$ | fluid |
| $s$ | solid |

coefficient is used at their interface anymore. Few papers can be found yet for ICFCP, see [10] for example, since articles on the corresponding direct problem only date back to the early 2000s, see [11–14]. However, to our knowledge, no work on ICFCP can be found with inversion not only of simulated measurements but also of real ones.

Our paper is organized as follows: in Section 2, we introduce the studied system (here the heat extractor) and its modelization. In Section 3, we derive the methodology for estimating the internal conditions from measurements over one of the external faces as well as the corresponding transfer function in a mini-heat extractor. In Section 4, we validate the methodology of Sections 2 and 3 using synthetic profiles generated by COMSOL. In Section 5, we apply this proposed methodology to a real experiment and will show the corresponding results.

## 2. The studied system and its transient modeling

### 2.1. The studied system

Let us consider a laminar fluid flow in a channel of length $2\ell$, of thickness $e_f$, limited by two parallel plates of thicknesses $e_1$ and $e_2$, see Fig. 1. The velocity profile $u(y)$ is assumed to be parabolic (Poiseuille flow) and fully developed from the inlet to the outlet of channel. The two solid layers (walls) and the fluid layer are characterized by their thermal conductivity $\lambda_i$, their volumetric heat $\rho c_i$ and their thermal diffusivity $a_i = \lambda_i/\rho c_i$ where $i = s_1, s_2$ or $f$ respectively.

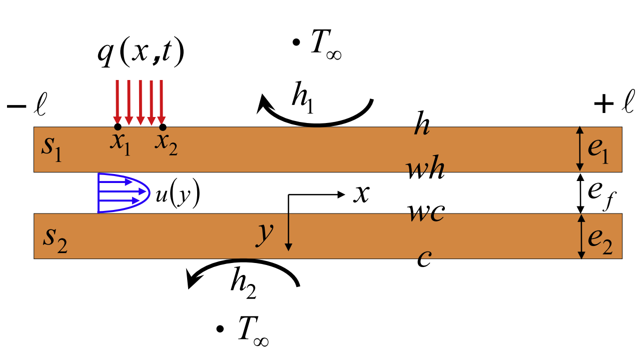


**Fig. 1.** $2D$ model of flow and heat transfer in the flat channel.

A surface heat source, $q(x,t)$ is imposed between $x_1$ and $x_2$ on the lateral hot face noted here $h$. The two lateral faces (hot $h$ and cold $c$) exchange heat with its surrounding environment (here ambient air) which is at an uniform temperature $T_\infty$. These exchanges are characterized by coefficients $h_1$ and $h_2$ respectively. These ones are assumed to be uniform on each face (they integrate natural convection and linearized radiation).

### 2.2. Modelization

The heat equation describing $2D$ heat transfer in transient state in the walls (upper wall, $s_1$ and lower wall, $s_2$), in the fluid layer ($f$) and the corresponding boundary and initial conditions, are:

- Heat equation in the solid (wall):

$$\frac{\partial^2 T_{s_i}}{\partial x^2} + \frac{\partial^2 T_{s_i}}{\partial y^2} = \frac{1}{a_{s_i}} \frac{\partial T_{s_i}}{\partial t} \quad \text{with} \quad i \equiv 1 \text{ or } 2 \tag{1}$$

- Heat equation in the fluid:

$$\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2} - \frac{u(y)}{a_f} \frac{\partial T_f}{\partial x} = \frac{1}{a_f} \frac{\partial T_f}{\partial t} \tag{2}$$

- External in-plane boundary conditions:

$$\varphi_h(x,t) = q(x,t) - h_1(T - T_\infty) \quad \text{at} \quad y = -\frac{e_f}{2} - e_1 \tag{3}$$

$$\varphi_c(x,t) = -h_2(T - T_\infty) \quad \text{at} \quad y = +\frac{e_f}{2} + e_2 \tag{4}$$

where $\varphi_h(x,t)$ and $\varphi_c(c,t)$ are the heat fluxes in the $y$ direction, on the $h$ and $c$ faces respectively. $q$ is the surface density of the heat source power. We assume here that $q(x,t)$ is separable and can be written as the product of a transient intensity $Q(t)$ (in W) by a space distribution $f(x)$ (in m$^{-2}$):

$$q(x,t) = Q(t)\, f(x) \tag{5}$$

- The solid/fluid interface conditions: at the solid/fluid interfaces, we assume the continuity conditions of heat flux and temperature:

$$-\lambda_{s_i} \frac{\partial T_{s_i}}{\partial y} = -\lambda_f \frac{\partial T_f}{\partial y} \quad \text{and} \quad T_{s_i} = T_f \quad \text{at} \quad y = \pm\frac{e_f}{2} \tag{6}$$

where $i \equiv 1$ if $y = -e_f/2$ and $i \equiv 2$ if $y = +e_f/2$