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Electro-thermo-convective flow of a dielectric liquid due to nonautonomous injection of charge by an elliptical electrode

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ABSTRACT

Electro-thermo-hydrodynamic (ETHD) flow induced by the simultaneous Coulomb and buoyancy forces in a dielectric medium is studied using the lattice Boltzmann method. Nonautonomous charge injection from a high temperature inner elliptical electrode to two cold parallel-plate electrodes is considered. Systematical simulations are conducted for ETHD problems with different injection models and non-dimensional parameters, including electric Rayleigh number T, Rayleigh number Ra and the ellipticity e of the elliptical electrode. It is found that the charge transport process, flow instability and heat transfer enhancement are significant affected by nonautonomous injection, especially in the Coulomb force dominant flow regime and the large ellipticity cases. Quantitatively, for dielectric liquid $M = 10$, $e = 2$ and $Pr = 10$ under strong injection $C = 10$, when compared to the autonomous charge injection assumption, nonautonomous injection shown an average of 16.4% increases in the mean Nusselt number within the range of driving parameters explored ($10^3 \leq Ra \leq 10^7$, $300 \leq T \leq 1800$).

1. Introduction

Electro-thermo-hydrodynamic (ETHD) flow induced by the simultaneous action of a unipolar injection of ions and a thermal gradient in dielectric liquids is the foundation of applications related to the transfer of heat, momentum and charge in these liquids [1,2]. In recent years, the ETHD problem has been extensively studied and recognized as a high energy efficiency technology in thermo-fluid systems, such as ETHD convection system [3], ETHD boiling and condensing systems [4], ETHD drying and evaporating systems [5], and ETHD solar energy systems [6]. As reported in recent reviews [1,7], most studies in ETHD are experimental, while numerical works are limited to simple physical models or simple geometries due to the complex mathematical model and the strong nonlinear coupled equations [2]. To gain fundamental insights into many poorly understood ETHD phenomena, more numerical studies should be devoted to ETHD.

The numerical methods for ETHD problems can be mainly classified into two categories. The first kind of methods are based on the conventional partial differential equations (PDEs), such as the finite difference method (FDM) [8], the finite element method (FEM) [9], and the finite volume method (FVM) [3,10]. This kind of methods has been successfully applied to most of ETHD problems both in two parallel-planes electrodes model [3] and annulus configuration [11,12]. However, due to the strongly convection-dominant feature of charge conservation equation, PDEs based methods need additional techniques to obtain accurate solution without unphysical oscillation. Some examples include the particle-in-cell scheme [13], the flux-corrected transport scheme [14], the total variation diminishing or high-resolution scheme [15]. The second kind is the particles based methods, such as the dissipative particle dynamic (DPD) [16] and the lattice Boltzmann method (LBM) [17–19]. Owing to the mesoscopic origin, this kind of methods can naturally track the transient evolution of fluid field, but their general drawback is time consuming. Among them, the recently developed LBM has a relatively high computing efficiency owing to its simple collision-streaming calculation process and intrinsic parallelism, even compared to macroscopic PDE-based methods. Besides, the LBM is proved to have second order accuracy [18] and strong flexibility for complex geometries [20].

Most of previous numerical works on electro-convective or electro-thermo-convective flows concerned the parallel-plane electrode configuration, this configuration has been widely used for the validation of theoretical model and numerical code [21–23]. However, curved or sharp electrodes which leads to local high electric fields and EHD plume are of great interest in applications such as ETHD enhanced heat transfer [7]. In that situation, the autonomous injection assumption with constant injection

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strength $q = C$ is inconsistent with real situations. Instead, a nonautonomous injection model with the charge density linked to the local electric field $q = f(E)$ at curved electrode surface should be adopted. Several nonautonomous injection models have been used for pure electro-convective flows by different authors [2,24,25] and in the excellent review [26]. Three representative cases are the linear injection model (LIM), the exponential injection model (EIM) and the injection law model (ILM), among them, the ILM expressed as $q_i = q_{io}/bK_1(b)$ (K_1 is the modified Bessel function of second kind and order one) is reported to be the most accurate one [2].

In this work, the configuration of an elliptical electrode between two parallel plates is used. The key motivation for our choice of the elliptical shape lies in the fact that the more curved elliptical surface corresponds to stronger charge injection strength and nonautonomous injection effects, which may lead to stronger flow field and the consequent increase of heat transfer rate. Although few published works in ETHD use elliptical electrodes, the related configurations containing circular cylinders have been extensively used [10,12,20]. By the forging technique, the cylindrical electrodes in these cases can be easily manufactured into the elliptical shape. Besides, some applications about the periodically arranged elliptical electrodes with pure EHD problems have been found, for example, elliptical metal posts in induced-charge electro-osmosis (ICEO) problem [27] and elliptical disc electrodes in electrochemical [28]. And in thermal convection problems, such as, the bank of tubes heat transfer systems [29], solar energy systems [30], and electronics cooling [31], and so on.

Therefore, ETHD flow under nonautonomous charge injection is simulated by solving the fully coupled governing equations using the lattice Boltzmann method. The main purpose of the present investigation is threefold: (i) to numerically investigate the effects of different injection models on heat transfer, fluid flow and charge density distribution (ii) to explain the formation of ETHD plume and to determine the flow transition from steady to unsteady states; (iii) to investigate the effect of curvature of elliptical electrode on heat transfer enhancement under nonautonomous injection.

2. Physical model and macroscopic governing equations

Consider periodically arranged elliptical cylinders between two parallel plates, and a single periodic unit cell of two dimensions (2D) is used for simulation as illustrated in Fig. 1. The aspect ratio of the configuration is $A = L/H = 0.5$, where H is the distance

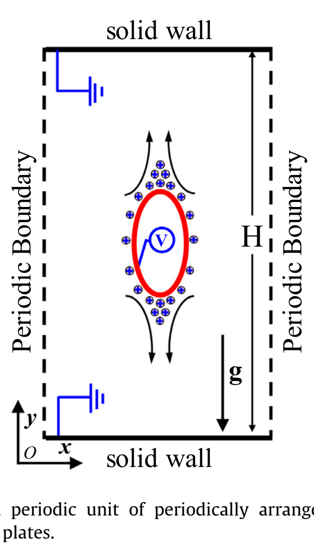

Fig. 1. Sketches of a periodic unit of periodically arranged elliptical cylinders between two parallel plates.

between the two plates and L is the length of the solid wall. The inner elliptical electrode with the major and minor axes being b and a (define $e = b/a$ as the ellipticity, $a/H = 0.1$) is kept at a constant electric potential ϕ_0 (>0) and high temperature θ_h. However, high curvature at the tip of ellipse provides a rapidly increasing of the local electric field E_c which leads to strong nonuniform distribution of local charge density distribution q according to the law $q = f(E_c)$. The free charges are then convected by the liquid and drifts relative to the flow motion with a velocity proportional to the electric field, until they reach the collecting electrode which is grounded $\phi_1 = 0$ with a low temperature θ_c. In the process, a Coulomb force caused by electric potential difference $\Delta\phi = \phi_0 - \phi_1$ and a thermal buoyancy force due to temperature difference $\Delta\theta = \theta_h - \theta_c$ simultaneously act on fluid and formulate the ETHD convection.

2.1. Governing equations and the unified lattice Boltzmann model

The mathematical equation of ETHD problems includes the mechanical equations, the electrical equations and the energy equation, a total number of six fully coupled nonlinear equations. Considering an incompressible, Newtonian and linear isotropic fluid under the Boussinesq approximation, the complete set of macroscopic governing equations are given as [32]

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0 \tag{1a}$$

$$\frac{\partial (\rho \mathbf{u})}{\partial t} + \nabla \cdot (\rho \mathbf{u}\mathbf{u}) = -\nabla \hat{p} + \nabla \cdot (\mu \nabla \mathbf{u}) + \mathbf{f}_b \tag{1b}$$

$$\nabla^2 \phi = -q/\varepsilon \tag{1c}$$

$$\mathbf{E} = -\nabla \phi \tag{1d}$$

$$\frac{\partial q}{\partial t} + \nabla \cdot \mathbf{j} = 0, \quad \mathbf{j} = (K\mathbf{E} + \mathbf{u})q - D\nabla q \tag{1e}$$

$$\frac{\partial \theta}{\partial t} + \mathbf{u} \cdot \nabla \theta = \nabla \cdot (\chi \nabla \theta) \tag{1f}$$

$$\begin{aligned}\mathbf{f}_b &= \mathbf{f}_t + \mathbf{f}_e \\ &= \rho_0[1 + \beta(\theta - \theta_{\mathrm{ref}})]\mathbf{g} + q\mathbf{E} - \frac{E^2 \nabla \varepsilon}{2} + \nabla\left[\frac{E^2}{2}\rho\left(\frac{\partial \varepsilon}{\partial \rho}\right)_\theta\right]\end{aligned} \tag{1g}$$

where $\mathbf{u} = [u, v]$ and $\mathbf{E} = [E_x, E_y]$ are fluid velocity field and electric field, respectively. The variables $\rho, \hat{p}, \phi, q$ and θ denote the fluid density, a modified pressure [33], electric potential, charge density and temperature; The symbols $\mu, \beta, \varepsilon, K, D, \chi$ stand for the dynamic viscosity, the coefficient of volumetric expansion, electrical permittivity, ionic mobility, charge-diffusion coefficient and thermal diffusivity. The body force $\mathbf{f}_b$ in Eq. (1g) consists of a thermal buoyancy force $\mathbf{f}_t$ and an electrical force $\mathbf{f}_e$ [33]. The ionic mobility K and electrical permittivity ε are assumed to vary linearly with temperature. Then, we have the equations of state [34]

$$K = K_0[1 + k_1(\theta - \theta_{ref})], \quad \varepsilon = \varepsilon_0[1 + e_1(\theta - \theta_{ref})]$$

in which, the subscript "0" denotes values at the reference temperature, k_1 and e_1 are derivatives with respect to temperature. Considering the ETHD phenomena are affected by many factors, in this work we mainly focused on the nonautonomous injection effect and the condition $k_1 = 0$, $e_1 = 0$ is used. Therefore, the last two terms in Eq. (1g), the dielectric force and the electrostrictive force respectively, can be neglected. Moreover, magnetic phenomena and the Joule effect are disregarded here.