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Natural convection in a square enclosure with different positions and inclination angles of an elliptical cylinder Part I: A vertical array of one elliptical cylinder and one circular cylinder

Seong Hyun Park [a], Yeong Min Seo [a], Man Yeong Ha [a], Yong Gap Park [b,*]

[a] School of Mechanical Engineering, Pusan National University, Jang Jeon 2-Dong, Geum Jeong Gu, Busan 609-735, Republic of Korea
[b] Rolls-Royce and Pusan National University Technology Centre in Thermal Management, Jang Jeon 2-Dong, Geum Jeong Gu, Busan 609-735, Republic of Korea

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ABSTRACT

This paper numerically investigates the two-dimensional natural convection in a square enclosure with a vertical array of a hot elliptical cylinder and a hot circular cylinder with Rayleigh numbers in the range of $10^4 \leqslant Ra \leqslant 10^6$. An immersed boundary method was used to capture the wall boundary of the cylinders. The effects of the position and inclination angle ϕ of the elliptical cylinder were investigated. When the Rayleigh number increases to $Ra = 10^6$, the numerical solutions reach an unsteady state for all cases of the lower elliptical cylinder and the cases of the upper elliptical cylinder except at $\phi = 90^\circ$. The transition of the flow regime from unsteady state to steady state depends on the flow direction and the space between the upper cylinder and top wall of the enclosure due to the changes in the inclination angle of the elliptical cylinder. At $Ra = 10^6$ in the case of upper elliptical cylinder at $\phi = 0^\circ$, the time- and surface-averaged Nusselt numbers for the walls of the enclosure increase by about 1.99% compared to the case of two circular cylinders. The thermal performance and flow stability were influenced by the position and inclination angle of the elliptical cylinder.

1. Introduction

Natural convection induced by hot objects in an enclosure is relevant to many industrial and environmental applications, such as heat exchangers, nuclear reactors, electronic equipment cooling, and stratified atmosphere boundary layers. In the past several decades, much research has been carried out to analyze the natural convection numerically [1–10]. Most of the studies have focused on the natural convection with a circular cylinder in a square enclosure with different boundary conditions and locations of the cylinder. However, few studies have been conducted with an elliptical cylinder in an elliptical enclosure [11–13] or square enclosure [14–17].

Tayebi et al. [11] conducted a numerical study on the natural convection in an annulus between two confocal elliptic cylinders filled with a hybrid nanofluid with various volume fractions of nanoparticles and Rayleigh numbers. Bouras et al. [12] studied the phenomenon of natural convection in a space annulus situated between two horizontal confocal elliptic cylinders. They analyzed the effect of the Reyleigh number and Prandtl number without regard to the variation of the numerical domain. Mahfouz [13] numerically investigated the buoyancy driven flow and associated heat convection in an elliptical enclosure using the Fourier spectral method. The inner surface of an elliptical tube was heated and maintained at either uniform temperature or uniform heat flux. Various Rayleigh numbers, tube axis ratios, and angles of inclination were used. The heat convection process in the enclosure depends on the geometry of the enclosure and the angle of inclination with respect to the gravity vector.

Raman et al. [14] conducted a numerical study on the natural convection over a heated elliptic cylinder at the center of a square cavity with cooled walls. They investigated the effect of the axis ratio of the elliptical cylinder at different Rayleigh numbers and two different aspect ratios of the cavity. The surface-averaged Nusselt number at the elliptical cylinder increased with increasing axis ratio of the elliptical cylinder and Rayleigh number and with decreasing cavity ratio.

Zhang et al. [15] conducted a numerical simulation for steady-state natural convection in a cold square enclosure containing a hot elliptical cylinder using the variational multiscale element-free Galerkin method (VMEFG). The numerical calculations were performed for various Rayleigh numbers of 10^3 to 10^6, inclination angles of the outer square enclosure of 0° to 45°, and major axis

* Corresponding author.
E-mail address: pyg777@pusan.ac.kr (Y.G. Park).

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Nomenclature

Symbols

a	semi-major axis
b	semi-minor axis
f_i	momentum forcing
g	acceleration of gravity
L	length of square enclosure
n	normal direction to the wall
Nu	local Nusselt number
$\langle Nu \rangle$	surface-averaged Nusselt number
$\overline{\langle Nu \rangle}$	time and surface-averaged Nusselt
P	dimensionless pressure
Pr	Prandtl number ($= \nu/\alpha$)
r	dimensionless radius of the cylinder ($= R/L$)
R	radius of circular cylinder
Ra	Rayleigh number $\left(= \frac{g\beta L^3 (T_h - T_c)}{\nu\alpha}\right)$
t	dimensionless time
θ	dimensionless temperature
u_i	dimensionless velocity
x_i	dimensionless Cartesian coordinates

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
δ_{i2}	Kronecker delta
ρ	density
ν	kinematic viscosity
ϕ	inclination angle of elliptical cylinder

Superscripts/subscripts

$*$	dimensional value
c	cold
h	hot
upper cyl	upper cylinder
En	enclosure
L	left wall
R	right wall
T	top wall
B	bottom wall

of the inner cylinder of 0.2 L to 0.4 L. The average Nusselt number on the inner elliptical cylinder increased with increasing Rayleigh number, but the distribution of these numbers was nearly independent of the tilt angle of the square enclosure.

Bararnia et al. [16] studied the natural convection between a square outer cylinder and a heated elliptical inner cylinder using the Lattice Boltzmann method. They investigated the fluid flow fields and isotherms for various vertical positions of the inner cylinder and Rayleigh numbers of 10^3 to 10^6. The position of the inner cylinder and Rayleigh number have remarkable effects on the streamlines, temperature contours, and vortex formation. The surface-averaged Nusselt number increases with the Rayleigh number, and the minimum value becomes more distant from the top of the inner cylinder with increasing Rayleigh number.

Liao et al. [17] carried out numerical investigations for natural and mixed convection within domains with stationary and rotating complex geometry using the immersed boundary method (IBM). They investigated the effect of the Rayleigh number, axis ratio, and inclination angle of an elliptical cross-section on the fluid flow fields and isotherms within a square enclosure. A higher inclination angle leads to higher average Nusselt numbers on both the cylinder and enclosure walls when the position of the cylinder is fixed. Previous studies reported that increases in the inclination angle of an elliptical cylinder and its major axis cause increases in the surface-averaged Nusselt number on both the cylinder and enclosure wall compared to a circular cylinder.

In recent years, many numerical studies have been conducted on the natural convection with multiple circular cylinders in a square enclosure [18–23]. A buoyant plume generated by each cylinder impinges on other cylinders in the enclosure, resulting in more complex isotherms and fluid flow fields. These complex phenomena result in different surface-averaged Nusselt numbers, fluid flow fields, and isotherms between a multi-cylinder array and a single cylinder. However, there is no information on the natural convection with multiple cylinders, including elliptical cylinders. Therefore, it is necessary to investigate the effects. In this study, we focus on the natural convection of a hot circular cylinder and a hot elliptical cylinder in a cold square enclosure in a vertical array with varying inclination angle of the elliptical cylinder.

2. Computational details

2.1. Geometry and boundary conditions

Fig. 1 indicates computational domain, coordinate system, and boundary conditions considered. The computational domain consists of a cold square enclosure with an inner hot elliptical cylinder and an inner hot circular cylinder. Two different cylinder arrays are considered with different locations of the elliptical cylinder, as shown in Fig. 1(a) and (b). The inclination angle of the elliptical cylinder was changed in the range of $0° \le \phi \le 90°$. The length of each wall of the enclosure is L, the radius of the circular cylinder is $R = 0.1L$, and the major and minor axes of the elliptical cylinder are $2a$ and $2b$ respectively.

The surface area of the elliptical cylinder is the same as that of the circular cylinder. The position of the centroid of each inner cylinder is fixed in all cases. The centroid of the upper cylinder is $(0, 0.25L)$, and that of the lower cylinder is $(0, -0.25L)$. Dimensionless temperatures are imposed on the cold enclosure walls ($T_c = 0$) and hot cylindrical surface ($T_h = 1$). No-slip and impermeability conditions are imposed on the surfaces of the cylinders and enclosure walls.

All the fluid properties are assumed to be constant except for the density in the buoyancy term. The Boussinesq approximation was used to model the variation of the fluid density in the buoyancy term due to the change in the fluid temperature. Gravitational acceleration is applied in the negative y-direction. The Prandtl number is 0.7, and the Reyleigh numbers are 10^4, 10^5, and 10^6. A grid resolution of 501×501 along the horizontal (x) and vertical (y) directions was used. Grid independence of the solution has been tested with additional simulations on much finer grids with up to 601×601 points. The differences in the results of the surface-averaged Nusselt numbers on the cylinder surface are less than 1%. Thus, the resolution of grid for the computational domain is sufficiently high. The numerical simulation code was validated by comparison with preliminary numerical simulations of the natural convection in a cold square enclosure with a hot cylinder considered by Bararnia et al. [16]. Table 1 shows the time- and surface-averaged Nusselt number at the enclosure wall in the Rayleigh number range of $10^4 \le Ra \le 10^6$. The time- and surface-