International Journal of Heat and Mass Transfer 126 (2018) 95–102

Contents lists available at ScienceDirect

International Journal of Heat and Mass Transfer

journal homepage: www.elsevier.com/locate/ijhmt

Linear stability of thermal-bioconvection in a suspension of gyrotactic micro-organisms

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ARTICLE INFO

Article history:
Received 21 August 2017
Received in revised form 27 March 2018
Accepted 5 May 2018

Keywords:
Convection
Linear stability
Suspension
Galerkin method

ABSTRACT

By utilizing a randomly swimming model, a linear stability analysis is applied to investigate the stability of bioconvection in a horizontal suspension layer of motile gyrotactic micro-organisms with heated from below. The micro-organisms under consideration are orientated by a balance between a gravitational torque, due to them being bottom heavy, and viscous torque arising from local fluid velocity gradients. The obtained eigenvalue problem containing thermal Rayleigh number and bioconvection Rayleigh number is solved numerically using one-term Galerkin method. The case of non-oscillatory instability is analyzed, the relationship among thermal Rayleigh number, bioconvection Rayleigh number, Lewis number, critical wavenumber and the shape of microorganisms are discussed. We point out that the heating from below makes the layer more unstable. When increasing the value of thermal Rayleigh number to 1750, the suspension becomes unstable itself, which imply that bioconvection Rayleigh number has nothing to do with the stability of this system. We also find that Lewis number has no effect on critical value of thermal Rayleigh number, but has a great influence on critical bioconvection Rayleigh number. The increasing cell eccentricity enlarges the critical value of bioconvection Rayleigh number, which means that the suspension is more stable.

1. Introduction

Micro-organisms, some of them being the oldest species in the world known to human beings, are very important for many things. For example, the global weather may be affected by algaes photosynthesizing in the sea and bacteria in our stomach could make the digestive system operate normally and systematically [1]. In the same manner as nature convection, the term 'Bioconvection' used to describe the phenomenon of gravitational overturning convection of micro-organisms. The up-swimming micro-organisms accumulate at the upper regions gradually, a gravitational overturning convection occurs due to the fluid has a lower density than the micro-organisms, when the cells concentration becomes sufficiently large. Pedley and Kessler [2] and Bill and Pedley [3] reviewed the work in this area, respectively.

Plesset and Winet [4] developed the first model of bioconvection in 1974. They investigated the preferred pattern wavelength which is the function of the upper layer depth and the cell concentration by using Rayleigh–Taylor instability in a continuously two-layer model. Then, a more convincing model was developed by Childress et al. [5], they assumed the micro-organisms could only swim upward for the asymmetric distribution of the micro-organisms, and they showed the appearance of the gravitational instability is analogous to Rayleigh-Bénard convection [6].

Many certain species of micro-organisms such as *Chlamydomonas nivalis* and *Euglena viridis* are structurally featured to be bottom-heavy. This has the consequence that a viscous torque is applied to the micro-organisms. There are a number of forces acting on the algae, the gravitational torque orienting the direction of cell and viscous torque originating from the shear flow, which leads to 'gyrotaxis' [7]. The effect of gyrotaxis is to tip the bottom-heavy cells away from regions of upflow and make the downflow regions denser than the upflow regions. The main forces affecting the cell are described in Fig. 1, assuming that the method of swimming has no effect on the flow field or the cell itself [8].

For the micro-organisms with asymmetric mass distributions in dilute suspensions, a new continuum model was formulated by Pedley [9]. It assumed that the cells swimming is random with direction, $\langle \mathbf{p} \rangle$, which satisfies the Fokker-Planck equation. Therefore, the mean cell orientation and the related translational diffusivity can be observed in a statistical manner. A good agreement

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https://doi.org/10.1016/j.ijheatmasstransfer.2018.05.030
0017-9310/

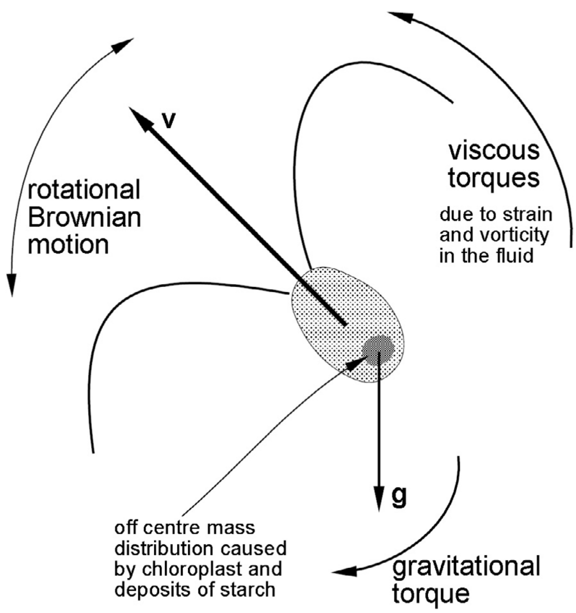

Fig. 1. The main forces affecting on the algae [11].

with the results observed in the experiment by Bees [10], which shows that this new continuum model is reasonable.

While the early studies focused mostly on suspensions in isothermal fluid, many micro-organisms are living in environment where temperature gradient exists, such as thermophiles micro-organisms which live in hot springs. Kuznetsov has studied the stability of the gyrotactic micro-organisms in a suspension heated form below, and showed the interaction between bioconvection and nature convection by using a linear stability analysis manner [12]. Bilgen and his coauthors investigated the effect of heating or cooling on the development of gravitactic bioconvection in vertical cylinders with stress free sidewalls [13,14], they observed the transition from a subcritical bifurcation to a supercritical bifurcation when the thermal Rayleigh number was increased. Nield and Kuznetsov [15] presented a linear stability analysis of a suspension of gyrotactic microorganisms in fluid layer of finite depth, and pointed out the cooling from below stabilizes the suspension. Sharma and Kumar [16] studied the linear stability of bioconvection in a dilute suspension of gyrotactic microorganisms in horizontal shallow fluid layer cooling from below and saturated by a porous medium. In spite of these interesting observations, there is no theoretical study that consider three different physical processes, randomly swimming, gyrotactic, and temperature gradient.

The aim of the present research is to study how temperature gradient effects the stability of a suspension of randomly swimming, gyrotactic micro-organisms. We consider a very simple situation in which a fluid layer heated form below has two constant temperatures at upper and lower surface, respectively. This study continues the research by Kuznetsov [12] and Bees [17] to investigate the relationship among bioconvection Rayleigh number, thermal Rayleigh number and wavenumber. Besides, the effects of other parameters such as the cell eccentricity and Lewis number on bioconvection Rayleigh number are considered, too.

The paper is organized as follow. In Section 2, we formulate the problem for a linear stability analysis. Some approximations made in the present study also are considered in this section. The solutions of basic state and many necessary parameters are presented in Section 3. The results of the linear stability are then given by using nondimensional parameters in Section 4. In Section 5, the Galerkin method is introduced to solve the eigenvalue value problem. A discussion about the obtained results is presented in Section 6, where the results of this paper are also compared with those of Rayleigh-Bénard convection and results given by Bees [17].

2. Problem formulation

Following the continuum model proposed by Kessler [18], we consider a water-based dilute suspension containing gyrotactic micro-organisms. The suspension is bounded by two infinitely wide and long parallel walls which are located at $z^* = 0$ and $z^* = H$, respectively. Here, z^* is relative direction of a cartesian coordinate system $Ox^*y^*z^*$ with the z^*-axis directed vertically upwards (asterisks denote dimensional variables). The suspension is assumed to be incompressible, we get

$$\nabla^* \cdot \mathbf{u}^* = 0, \tag{1}$$

where $\mathbf{u}^*$ is the velocity of the suspension. By considering the Boussinesq approximation and neglecting all effects of the cells except their negative buoyancy, the momentum equation can be expressed as

$$\rho \frac{D^*\mathbf{u}^*}{D^*t^*} = -\nabla^* p_e^* + \mu\nabla^{*2}\mathbf{u}^* + \rho\left(1 - \beta(T^* - T_c^*)\right)\mathbf{g} + n^*\upsilon\Delta\rho\mathbf{g}. \tag{2}$$

Here, t^* is the time, ρ is the density of suspension, p_e^* is excess pressure above the hydrostatic pressure, μ is the dynamic viscosity of the suspension (i.e., assumed to be approximately equal to water due to low concentration of micro-organisms), $\mathbf{g}$ is the gravity vector, β is the volumetric thermal expansion coefficient of water, n^* is the concentration of gyrotactic micro-organisms, υ is cells volume, $\Delta\rho$ is the density difference between the cell and water. T is the fluid temperature and the temperatures of the upper and lower wall are T_c and T_h, respectively.

The thermal energy equation is

$$\rho c\left[\frac{\partial T^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla^* T^*\right] = \nabla^*(k\nabla^* T^*), \tag{3}$$

where ρc is the volumetric heat capacity of water and k is the thermal conductivity of water.

Based on the continuum model [9,17], the governing equation for conservation mass can be described by the equation

$$\frac{\partial n^*}{\partial t^*} = -\nabla^* \cdot [n^*(\mathbf{u}^* + V_s\langle\mathbf{p}\rangle - \mathbf{D}^* \cdot \nabla^* n^*)], \tag{4}$$

where V_s is the mean cell swimming speed, $\mathbf{D}^*$ is the cell diffusion tensor (see [9] for details), and the mean cell direction $\langle\mathbf{p}\rangle$ is defined by

$$\langle\mathbf{p}\rangle = \int_S \mathbf{p}f(\mathbf{p})dS. \tag{5}$$

Here, S is the surface of the unit sphere and the diffusivity tensor $\mathbf{D}^*$ is approximated by the simplified expression given by Pedley and Kessler [9],

$$\mathbf{D}^* = V_s^2\tau\langle(\mathbf{p} - \langle\mathbf{p}\rangle)(\mathbf{p} - \langle\mathbf{p}\rangle)\rangle. \tag{6}$$

It should be noticed that some published work treated the diffusivity as a constant [12]. The value of τ is approximately equal 1.3 s, which can be measured in experiments [9,20], and $\mathbf{p}$ is a unit vector defined on the unit sphere,

$$\mathbf{p} = \begin{pmatrix} \sin\theta\cos\phi \\ \sin\theta\sin\phi \\ \cos\theta \end{pmatrix}, \tag{7}$$