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The effect of conducting bounding horizontal plates on species separation in porous cavity saturated by a binary mixture

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ABSTRACT

In this paper, an analytical and numerical study of species separation in binary mixtures taking account of the presence of bounding plates for the cell is presented. A rectangular horizontal porous cavity saturated by a binary mixture and heated from below is considered. This cavity is bounded by thin plates of uniform thickness, the outer surfaces of which are subjected to a constant heat flux. The transition from the equilibrium solution to the convective one, either stationary or oscillatory, was previously studied by Ouattara et al. (2012). Thus in the first part of this paper, the critical parameters associated with the onset of long wavelength disturbances, obtained analytically, are recalled. Then the hypothesis of parallel flow is used to determine an analytical solution which describes the unicellular flow which may appear in the case of a large aspect ratio cell for a given range of separation ratio, ψ, Rayleigh number, *Ra*, Lewis number, Le, the ratio of the plate to the porous layer thickness, δ, and their thermal conductivity ratio, *d*. The analytical results are corroborated by direct numerical simulations. We verify that if *d* goes to infinity, the walls become infinitely conductive and we find the results obtained by Charrier-Mojtabi et al. (2007). When *d* tends to 0, the walls become infinitely thin the results obtained by Yacine et al. (2016) are recovered.

A linear stability analysis of the unicellular flow is also presented. The eigenvalue problem resulting from the temporal stability analysis is solved by a Tau spectral method. The optimal Rayleigh number leading to an optimal value of the separation horizontal gradient is determined for different values of physical parameters. We show that the species separation depends sensitively on the ratio of the plate to the porous layer thickness, and the ratio of their thermal conductivities. Furthermore, we have shown that in the stationary state and for a given value of the thermal conductivity ratio ($d = 29$), the maximum separation is almost equal for walls of the same thickness than the one of porous cavity or for the case of porous cell delimited by the infinitely thin walls.

1. Introduction

In binary fluid mixtures subjected to temperature gradient, a mass fraction gradient appears due to the thermodiffusion or Soret effect. In addition to the usual expression for the mass flux $\vec{J}$ given by Fick's law, a part due to the temperature gradient is added so that:

$$\vec{J} = -\rho D \nabla C' - \rho D'_T \nabla T'$$

where D is the mass diffusion coefficient, ρ the density, and C' the mass fraction of the denser component. Here $D'_T = F(C')D_T$, where D_T is the thermodiffusion coefficient and $F(C')$ is a particular function of C' satisfying both $F(C' = 0) = 0$ and $F(C' = 1) = 0$. Most authors use the function, $F(C') = C'(1 - C')$ and make the assumption that $C'(1 - C') \approx C_0(1 - C_0)$ where C_0 is the initial value of the mass fraction.

Under the gravity field, the coupling between convection and thermo-diffusion, called thermo-gravitational diffusion, has been found to lead to species separation. Thermo-gravitational separation in a porous medium which is saturated by a binary mixture has been studied widely because of its numerous fundamental and industrial applications. Some examples of interest are the migration of moisture in fibrous insulation, the transport of

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Nomenclature

A	aspect ratio of the porous bulk $A = L/H$
a	effective thermal diffusivity of the porous-mixture system $a = \lambda_p/(\rho c)_f$
a_s	thermal diffusivity of metal $a_s = \lambda_s/(\rho c)_s$
C	mass fraction of denser component of the mixture
C_0	initial mass fraction of the denser component of the mixture
m	mass fraction gradient along the horizontal axis
m^r	real mass fraction gradient along the horizontal axis
d	thermal conductivity ratio $d = \lambda_s/\lambda_p$
D^*	mass diffusion coefficient (m^2 s^{-1})
D_T^*	thermodiffusion coefficient (m^2 s^{-1} K^{-1})
H	height of the porous layer (m)
h	height of the horizontal plates (m)
K	permeability of the porous medium (m^2)
k	wave number
k_c	critical wave number
L	length of the cavity (m)
Le	Lewis number $Le = a/D^*$
q'	uniform heat per unit area (W·m^{-2})
Ra	Darcy-Rayleigh number $Ra = (KHg\beta_T\Delta T)/(a\nu)$
Ra_c	critical Rayleigh number
t	nondimensional time (s)
T_1, T_3	temperatures inside the lower and upper plates (K)
T_2	temperature inside the porous bulk (K)
$\vec{\mathbf{V}}$	flow velocity (ms^{-1})
u, v	Horizontal and vertical velocity components (ms^{-1}

Greek symbols

β_T	thermal expansion coefficient (K^{-1})
β_C	solutal expansion coefficient
ε^*	porosity of porous medium
ε	normalized porosity of the porous medium
δ	ratio of the plate to the porous layer thickness $\delta = h/H$
ρ	density of the mixture (kg m^{-3})
ψ	separation ratio $\psi = -(\beta_C/\beta_T)(D_T^*/D^*)C_0(1 - C_0)$
ψ_{uni}	separation ratio beyond which flow at onset of convection is unicellular
λ_p	effective thermal conductivity of the saturated porous medium (W m^{-1} K^{-1})
λ_s	thermal conductivity of the horizontal plates (W m^{-1} K^{-1})
$(\rho c)_f$	volumetric heat capacity of the mixture (J m^{-3} K^{-1})
$(\rho c)_p$	effective volumetric heat capacity of saturated porous medium (J m^{-3} K^{-1})
ν	kinematic viscosity of mixture (m^2 s^{-1})
φ	stream function
θ_1, θ_3	temperature perturbations inside the lower and upper plates
θ_2	bulk temperature perturbation
α	thermal diffusivity ratio $\alpha = a_s/a$

Superscripts

$'$	dimensional variable

contaminants in saturated soil, drying processes or solute transfer in the mushy layer during the solidification of binary alloys.

Reviews of recent developments and publications in this field are given by Nield and Bejan [1], Ingham and Pop [2], Vafai [3] and more recently by Vadasz [4]. A compilation of the most pertinent information on the critical Rayleigh number and wavenumber associated to the onset of convection in an infinite porous layer saturated by a mono-constituent fluid with different boundary conditions, (i.e. free or impermeable; prescribed temperature or prescribed heat flux) may be found in Nield and Bejan [1].

Zebib and Bou-Ali [5] performed a linear stability analysis of a binary mixture buoyant return flow in a tilted, differentially heated, infinite layer using an asymptotic long-wave analysis and pseudo-spectral Chebyshev numerical solutions. For negative separation ratio, it was shown that longitudinal instabilities with small wave numbers are triggered at any finite temperature difference for all inclination angles except when the layer is close to the horizontal for either the heating-from-above or heating-from-below configurations. Numerical results were given for a specific water-ethanol mixture and were in good agreement with the asymptotic results. Transition from the longitudinal stationary instabilities in inclined layers to these instabilities in horizontal layers was also presented for this mixture.

With regard to thermo-gravitational separation, in 1938, Clusius and Dickel [6] successfully carried out the separation of gas mixtures in a vertical cavity heated from the side (thermo-gravitational column, TGC). Furry et al. [7] then developed the theory of thermodiffusion to explain the experimental process involved in isotope separation. For differentially-heated vertical columns, the authors showed that there is a maximum separation for an optimal value of the cell thickness. Subsequently, many works have followed, in order to justify the assumptions or extending the results of the FJO theory to the case of binary liquids. Lorenz and Emery [8] proposed to introduce a porous medium into the cavity in order to increase the width of the cell corresponding to the maximum separation.

Bennacer et al. [9] studied the double diffusive convection in a vertical annular porous medium subjected to a horizontal temperature gradient. An increase in the curvature of the cylindrical annulus permits a higher species separation due to the nonsymmetrical temperature profile. To overcome such limitations, two sub-domains allowing filtration separation were proposed and investigated. The separation ability increases with the partitioning number. In the same configuration, and for double diffusive convection without Soret effect, Marcoux et al. [10] showed the curvature effect on the temperature and mass fraction field. More recently Abahri et al. [11], studied the separation of a binary mixture occurring in a horizontal porous annular layer. The inner and outer cylinders were kept at different and constant uniform temperatures Ti and To, with Ti < To. They used the perturbation method by developing the temperature, stream function and mass fraction in terms of a power series in the Rayleigh number, to provide solutions for low Rayleigh number flow regimes. Direct numerical simulations, using the finite element method, were performed to corroborate the results obtained analytically.

Charrier-Mojtabi et al. [12] and Elhajjar et al. [13] presented an analytical and numerical stability analysis of Soret-driven convection in a porous cavity saturated by a binary fluid. The porous cavity is bounded by horizontal surfaces of either infinite or finite extent and it was heated either from below or from above. These horizontal plates were maintained at different but constant temperatures. From the linear stability analysis, the authors found that the equilibrium solution loses its stability via a stationary bifurcation or a Hopf bifurcation depending on the separation ratio and the normalized porosity of the medium. The role of the porosity is important: when it decreases; the stability of the equilibrium solution is reinforced. In