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Change in viscosity of Williamson nanofluid flow due to thermal and solutal stratification

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ABSTRACT

Current analysis is devoted to explore the computational aspects of variable viscosity on Williamson nanofluid over a non-linear stretching sheet. Viscosity of the fluid is assumed to be depend on temperature and due to thermal stratification, viscosity of the fluid also depends upon thermal diffusion. The basic mathematical problem (system of PDEs) is converted into non-linear ODEs via applying suitable transformations. Computational solutions of the problem is achieved by efficient numerical approach (shooting method). Characteristics of controlling parameters i.e. Lewis number, thermophoresis parameter, Hartmann number, plastic dynamic viscosity parameter, Weissenberg number, Prandtl number, stretching index, Brownian motion parameter, Prandtl number, thermal and solutal stratification parameters are plotted on concentration, velocity and temperature gradients. Furthermore friction factor coefficient, heat and mass diffusion rates are presented through graphs and tables. Conclusions are made on the basis of entire investigation and it is seen that velocity profile reduces for large values of variable viscosity and thermal stratification parameters while thermal stratification parameter shows opposite behavior for temperature profile. Moreover, concentration profile reduces for enhancing values of Lewis number and increases for large values of stretching velocity parameter.

1. Introduction

The study of heat transport is of great interest for researchers, engineers, designers, developers and manufactures due to its wide range of applications in petroleum reservoirs, chemical reactor catalytic, nuclear waste disposal, etc. Abel and Mahesha [1] examined the heat transfer characteristics of viscoelastic fluid flow over a stretchable sheet with thermal conductivity and non-uniform heat and source. Aleo et al. [2] discussed the thermo physical aspects of Newtonian fluid flow over a semi-infinite moving vertical sheet. Khan et al. [3] presented the variable thermal conductivity of unsteady squeezed Carreau fluid flow over a sensor surface. Mondal eta al. [4] inspected the influence of chemical reaction and inclined Lorentz forces on force and free convection fluid flow over an inclined stretching surface. Sreedevi et al. [5] studied the mass and heat diffusion fluid flow along with chemical reaction and thermal radiative heat flux over a nonlinear stretching sheet. Zhang et al. [6] measured the heat diffusion characteristics for unsteady power law nanofluid flow over a stretching plate with slip velocity effects and variable magnetic field. They illustrated that the behavior of Cuo nanomaterials have more distinguished impact on heat transfer enhancement rate. Khan et al. [7] presented the concept of axisymmetric second grade nanofluid flow with thermal diffusion over an isothermal nonlinear radially stretchable sheet. Jahan et al. [8] exemplified the heat assignment of nanoliquid flow past a porous nonlinear shrinking and stretching sheet. Bilal et al. [9] scrutinized the thermal conductivity change in heat transfer of Williamson bidirectional fluid flow over a nonlinear stretching surface under heat generation/absorption effects. Raju et al. [10] discussed simulations in convective heat transport Carreau fluid flow over a cone filled with several alloy nanomaterials. Ibrahim et al. [11] examined the combined effects of free and force convection Casson nanoliquid flow over a permeable surface under magnetic field and chemical reaction. Pal and Mandal [12] demonstrated the convective thermal diffusion effects in viscous nanofluid flow over a nonlinear stretching sheet with thermal stratification and viscous dissipation.

Nanoparticles have complete revolutionary modification in the field of liquid dynamics. Choi [13] considered the nanofluid in his work by inserting numerous metallic materials in liquids and found that nanoparticles have much better thermal properties than their base fluids. Malik et al. [14] described the 2-D similarity results of Casson nanomaterials flowing along a vertical cylinder.

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Nomenclature

B_0	magnetic field strength	C_0	reference concentration
U_0	reference stretching rate	T_0	reference temperature
m	stretching velocity index	T_w	wall temperature
n	power law index	C_w	nanoparticle fraction
ν	kinematics viscosity	C, C_∞	concentration susceptibility
k	thermal conductivity	T, T_∞	temperature of fluid
c_p	specific heat	u,v	velocity components along x-axis and y-axis
σ	electrical conductivity	θ	dimensionless temperature
$\frac{k}{\rho c_p}$	thermal diffusivity	ϕ	dimensionless concentration function
ζ	similarity variable	Γ	time fluid parameter
τ	effective heat capacity of the nanoparticle and heat capacity of the fluid	M	Hartmann number
		We	Weissenberg number
ρ	density	*Pr*	Prandtl number
μ	temperature based viscosity	N_b	Brownian moment
D_B	mass diffusivity	N_t	thermophoresis parameter
D_T	thermophoresis diffusivity	α	wall thickness parameter
b	physical parameter related to stretching sheet	*Le*	Lewis number
b_1	temperature dependent viscous parameter	S_t	thermal stratification parameter
b_2	temperature dependent thermal conductivity parameter	S_c	solutal stratification parameter
A	coefficient related to stretching sheet	C_f	skin friction coefficient
ς	temperature dependent viscosity parameter	Sh_x	local Sherwood number
Nu_x	local Nusselt number		

Sulochana and Sandeep [15] analyzed the MHD free and force convection nanomaterial fluid flow over a stretching cylinder in porous medium. Khan et al. [16] calculated the numerical results of combined force and free convection nanomaterial fluid flow between nonisothermal sheets. Basier et al. [17] deliberated the slip flow of Newtonian fluid flow comprising gyrotatic microorganisms past a stretching cylinder. Salahuddin et al. [18] exemplified the impact of MHD stagnation point flow over a hyperbolic tangent nanoliquid over a stretching surface. Thumma et al. [19] calculated the numerical solution of nanofluid flow along a nonlinear inclined stretching sheet.

Thermal and solutal stratifications plays a significant role in the progress of industries, engineering and sciences. The applications of stratification includes heat rejection system such as seas, rivers and lakes, polymer extrusion, thermal energy and condensers of power plants. Mukhopadhyay [20] examined the numerical solutions of MHD fluid flow over exponentially stretching surface along with thermal and solutal stratification. Mhamood et al. [21] exemplified the influence of Lorentz forces on Newtonian fluid flow over a stretching surface in thermally stratified medium. Rehamn et al. [22] examined the thermal and solutal stratification in double convection Williamson fluid flow over an inclined stretching cylinder. Daniel et al. [23] investigated the impact of double thermal stratification on MHD nanoliquid flow (Buongiorno's model) due to nonlinear stretching sheet.

Previously, Ajayi et al. [24] examined double stratification and variable viscosity effects on Casson fluid flow over a stretching sheet having variable thickness along with viscous dissipation. Motivated by this work, our main theme is to discuss the double stratification and variable viscosity effects on Williamson fluid flow over a non-linear stretching sheet. Moreover, in order to increase the thermal conductivity of Williamson fluid nanoparticles are added. Magnetic field is applied normal to the plate.

2. Mathematical formulation

Let us consider a mathematical model for two dimensional boundary layer flow of Williamson nanofluid flow over a continuously nonlinear stretching surface with variable dynamic viscosity. The plate is stretched with velocity $U_w = U_0(x+b)^{m-1}$. The surface is taken at $y = A(x+b)^{\frac{1-m}{2}}$, here A is very small constant, m is the stretching index, U_0 is the stretching rate and b is the dimensionless constant. Further it is assumed that the model must be fulfilled only for $m \neq 1$, because for $m = 1$, it reduces to flat surface. A uniform external magnetic field B_0 is applied in normal direction of flow, vertical to the sheet (as illustrated in Fig. 1).

Under these assumptions and boundary layer approximations the governing equations (Williamson nanofluid model, energy and concentration equations) become:

$$\partial_x u + \partial_y v = 0, \tag{1}$$

$$u(\partial_x u) + v(\partial_y u) = \frac{1}{\rho}\partial_y(\mu(T)\partial_y u) + \frac{\Gamma\sqrt{2}}{\rho}\left[\partial_y(\mu(T)\partial_y u)\frac{\partial u}{\partial y}\right] - u\frac{\sigma B_0^2}{\rho}, \tag{2}$$

$$u\partial_x T + v\partial_y T = \tau\left\{(D_B\partial_y C)\partial_y T + \frac{D_T}{T_\infty}(\partial_y T)^2\right\} + \frac{k}{\rho c_p}(\partial_{yy}T), \tag{3}$$

$$u\partial_x C + v\partial_y C = \left[D_B(\partial_{yy}C) + \left(\frac{D_T}{T_\infty}\right)\partial_{yy}T\right], \tag{4}$$

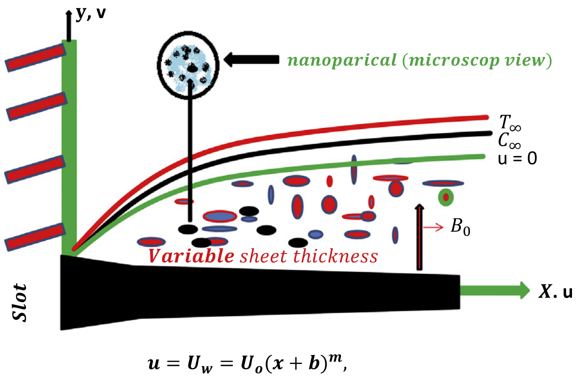

Fig. 1. Geometry of the problem.